Chapter 12

Exercise 12A

1 a $y = 2x + 4$

b $y = 3x - 2$

c $y = -x + 6$

d $y = -5x - 2$

2 a $y = \frac{2}{3}x - 2$

b $y = \frac{3}{5}x + 2$

c $y = -\frac{1}{3}x - 1$

d $y = -\frac{5}{2}x + 1$

3 a $y = 3x + 2$

b $y = \frac{1}{3}x + 4$

c $y = 5x - 5$

d $y = \frac{3}{4}x + 6$

e $y = -3x + 8$

f $y = -x - 2$

g $y = -\frac{2}{3}x + 5$

h $y = -\frac{3}{4}x - 3$

i $y = \frac{4}{5}x + 4$

4 a $C = \frac{7}{5}m + 1.90$

b Each of them will have to pay £29.50.

c John travelled 30 miles.

5 a $a = 208,\ b = \frac{7}{10}$

b If you substitute t=55 years in the equation $R = 208 - \frac{7}{10}t$ you will find R=169.5bpm, so Lorna should slow down.

6 a $p = \frac{9}{5},\ q = 32$

b Temperature = 86 °F

c Temperature = −89.2 °C

Exercise 12B

1 a $y - 1 = 2(x - 6)$

b $y + 8 = 5(x - 3)$

c $y - 5 = -4(x + 1)$

d $y + 9 = \frac{1}{3}(x + 2)$

e $y - d = t(x - c)$

2 a $y = 3x - 13$

b $y = 2x - 2$

c $y = -8x + 13$

d $y = \frac{1}{2}x - 5$

e $y = -\frac{2}{3}x - \frac{26}{3}$

f $y = -\frac{3}{5}x + \frac{38}{5}$

g $y = tx + d - ct$

3 a $y - 7 = x - 2$ or $y - 10 = x - 5$

b $y - 2 = \frac{9}{5}(x + 1)$ or $y - 11 = \frac{9}{5}(x - 4)$

c $y - 1 = 2(x + 5)$ or $y - 13 = 2(x - 1)$

d $y + 4 = 2(x - 3)$ or $y + 12 = 2(x + 1)$

e $y + 7 = -\frac{1}{2}(x - 9)$ or $y + 2 = -\frac{1}{2}(x + 1)$

f $y - 8 = \frac{3}{5}(x + 8)$ or $y - 11 = \frac{3}{5}(x + 3)$

4 a $y = \frac{1}{2}x + \frac{1}{2}$

b $y = 4x - 4$

c $y = -x + s + t$

d $y = \frac{1}{2}x + \sqrt{2}$

5 a $w = \frac{19}{10}h - 62$

b The formula can't be used for men who are particularly short, because the weight would be negative.

6 $y_{AC} = -5x + 16$

7 $m = \frac{c^2-d^2}{c-d}$ that can be simplified as $m = c + d$. From $y - c^2 = (c + d)(x - c)$ or $y - d^2 = (c + d)(x - d)$ you will get $y = (c + d)x - cd$

Exercise 12C

1 a $f(4) = 18,\ f(0) = -2,\ f(-2) = -12$

b $b = 3$

c $x = 3$

2 a $g(5) = -9,\ g(-3) = 7,\ g\left(\frac{2}{3}\right) = -\frac{1}{3}$

b $p = -5$

3 a $f(-6) = -9,\ f\left(\frac{3}{4}\right) = -\frac{9}{2}$

b $p = 9$

4 a $f(3) = -8,\ f(0) = -5,\ f(-2) = 7$

b $x_1 = -1,\ x_2 = 5$

c $x_1 = x_2 = 2$

d $f(x) = -9$ at the turning point of the parabola.

5 a $h(2) = 180,\ h(-3) = -345$

b $t = 0,\ 20$

6 a $f(3) = 16,\ f(-1) = -4$

b $f\left(\frac{1}{2}\right) = -\frac{17}{8}$

7 $x = \frac{4}{3}$

8 $x = -\frac{1}{3}$

9 a $p(x) - q(x) = \frac{5x-2}{12}$

b $x = \frac{2}{5}$

10 a $f(3) = 64,\ f(-2) = \frac{1}{16},\ f\left(\frac{3}{2}\right) = 8$

b $x = -\frac{1}{2}$

Exercise 12D

1 a $y = -4x - 2$

b $y = -\frac{3}{2}x + \frac{5}{2}$

c $y = 5x + 2$

d $y = -\frac{2}{7}x + \frac{4}{7}$

e $y = -6x - 12$

f $y = \frac{5}{4}x + 5$

2 a $m = -4$; $(0, -2)$

b $m = -\frac{3}{2}$; $\left(0, \frac{5}{2}\right)$

c $m = 5$; $(0, 2)$

d $m = -\frac{2}{7}$; $\left(0, \frac{4}{7}\right)$

e $m = -6$; $(0, -12)$

f $y = \frac{5}{4}$; $(0, 5)$

3 a i: $(0, -8)$

ii: $(4, 0)$

b i: $(0, 10)$

ii: $(-2, 0)$

c i: $(0, 5)$

ii: $\left(\frac{5}{3}, 0\right)$

d i: $(0, -3)$

ii: $(6, 0)$

e i: $(0, 9)$

ii: $(-15, 0)$

f i: $(0, -16)$

ii: $(-12, 0)$

4 a i: $(0, 6)$

ii: $(6, 0)$

b i: $(0, -2)$

ii: $\left(\frac{1}{2}, 0\right)$

c i: $(0, -2)$

ii: $(4, 0)$

d i: $\left(0, \frac{9}{2}\right)$

ii: $(-12, 0)$

e i: $(0, -4)$

ii: $(6, 0)$

f i: $(0, -16)$

ii: $(-12, 0)$

g i: $(0, 15)$ **i:** $(5, 0)$

h i: $(0, 3)$ **i:** $(4, 0)$

i i: $(0, 24)$ **i:** $(-36, 0)$

j i: $(0, 12)$ **i:** $(10, 0)$

k i: $(0, -8)$ **i:** $(6, 0)$

l i: $(0, 0)$ **i:** $(0, 0)$

5 $y = \frac{1}{3}x - 3$

6 $y = -\frac{3}{4}x + \frac{7}{2}$

7 $P(8,0)$; $Q(0,-3)$

$Area = 12$ square units

8 a $S(11, 6)$

b i: $y_{QS} = -2x + 28$

ii: QS intercepts the y-axis in $(0, 28)$

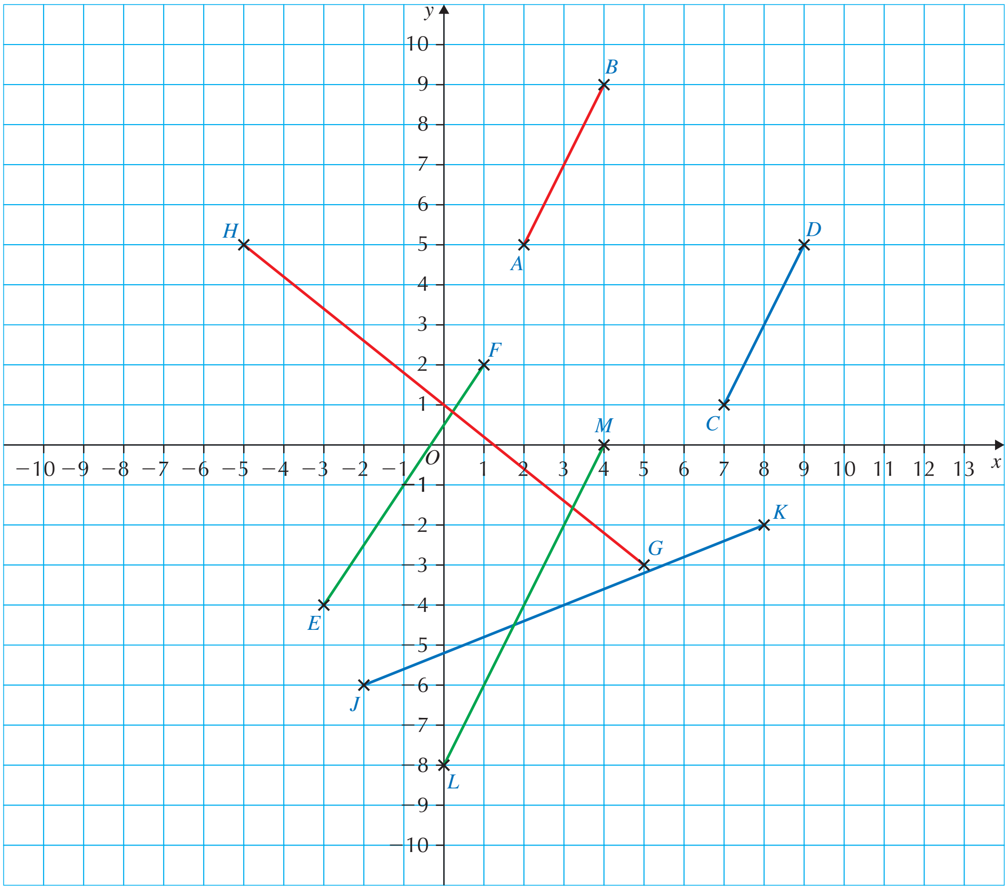
c $M\left(\frac{10}{3}, 0\right)$

9 $P\left(0, \frac{11}{2}\right)$; $Q\left(0, \frac{23}{2}\right)$; $M\left(4, \frac{17}{2}\right)$

$A_{MPQ} = 12$ square units

Activity p. 109

a



b $M_{AB}(3, 7)$; $M_{CD}(8, 3)$; $M_{EF}(-1, -1)$; $M_{GH}(0, 1)$; $M_{JK}\left(\frac{7}{2}, -4\right)$; $M_{LM}(2, -4)$

c Pupil's own answers. May suggest taking average of the two x-coded and the two y-words $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Exercise 12E

1 a $midpoint_{PQ}(4, 3)$

b $midpoint_{AB}(3, 7)$

c $midpoint_{CD}(-7, -3)$

d $midpoint_{RS}\left(\frac{11}{2}, \frac{5}{2}\right)$

e $midpoint_{VU}\left(\frac{u+v}{2}, \frac{v+u}{2}\right)$

f $midpoint_{GH}\left(\frac{3g}{2}, \frac{3h}{2}\right)$

2 $midpoint_{AC}\left(\frac{11}{2}, \frac{3}{2}\right)$; $midpoint_{BD}\left(\frac{11}{2}, \frac{3}{2}\right)$

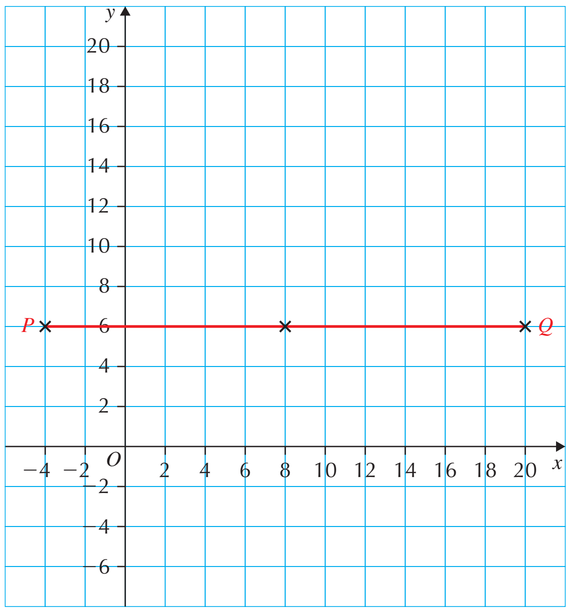
3 a $x_{QS} = 8$

b $(8, -3)$

4 $midpoint_{PQ}\left(\frac{a+b}{2}, \frac{a^2+b^2}{2}\right)$

$m_{PQ} = a + b$, L is perpendicular to PQ,

so $m_L = \frac{1}{a+b}$, and from here you can derive the equation of L passing through M.



Chapter 13

Exercise 13A

1 **a** $x = 3\frac{1}{2}$

b $x = -3\frac{1}{3}$

c $x = -1\frac{1}{2}$

d $x = \frac{3}{8}$

e $x = -17$

f $x = 1\frac{1}{3}$

2 **a** $x = 3$

b $x = 4$

c $x = -3$

d $x = 7$

e $x = 10$

c $x = -3$

d $x = 7$

e $x = 10$

f $x = 3$

3 **a** $x = 3$

b $x = 6$

c $x = -\frac{19}{40}$

d $x = 4$

e $x = -2$

f $x = 1$

g $x = 4$

h $x = -7\frac{4}{9}$

4 **a** $x = 1\frac{1}{2}$

b $x = -2$

c $x = 4$

d $x = -2\frac{3}{5}$

e $x = \frac{4}{5}$

f $x = -\frac{1}{2}$

g $x = 2\frac{3}{7}$

h $x = 6\frac{1}{2}$

5 171 cm^2

6 **a** if x is the number that Kyle and Seonaid were given then $6x - 4 = 4(x + 3)$. Giving $x = 8$

b 44

7 Deidre scored 28 in game 1, 134 in game 2 and 405 in game 3.

Activity p. 114

An athlete in the inside lane should run 0.30 m from the edge of the track.

Exercise 13B

1 a $x = 30$
b $x = -36$
c $x = 8$
d $x = 14$
e $x = 40$
f $x = 54$
g $x = 24$
h $x = 7$
i $x = -3$

2 a $x = 6\frac{2}{3}$
b $x = 24$
c $x = \frac{6}{11}$
d $x = -30$
e $x = 44\frac{4}{9}$
f $x = -3\frac{1}{5}$

3 a $x = 9$
b $x = 49$
c $x = \frac{13}{17}$
d $x = -1\frac{18}{23}$
e $x = 3$
f $x = 7\frac{4}{7}$
g $x = -\frac{11}{16}$
h $x = \frac{2}{13}$

4 $x = 72$ biscuits.

5 Original number is 9.

Exercise 13C

1 a $x > 3$
b $x < 7$
c $x > -5$
d $x < 2$
e $x < 2$
f $x \le 1$
g $x > -5$
h $x < \frac{1}{3}$
i $x < 4$

2 a $x > 1\frac{1}{2}$
b $x > -\frac{1}{5}$
c $x \le 5$
d $x < \frac{1}{2}$
e $x < -\frac{5}{4}$
f $x \le -\frac{1}{2}$
g $x \le 4$
h $x > -2$
i $x > -3\frac{1}{2}$
j $x > -2$
k $x < 6$
l $x < -\frac{1}{2}$
m $x > -5$
n $x \ge -5$

3 a Pukka plumbing $C = 60 + 25h$
b Perfect Plumbing $C = 25 + 35h$
c Student's answer may vary depending on how long it is estimated to fix the problem. Assuming that it will take less than 3.5 hours for a plumber to fix the problem, Kyle should call Pukka Plumbing as they will be cheaper.

4 a No the claim is not justified. The actual cost is £3.80 for the first mile and £1.60 per mile after that.
b Local authority B: £3.25 for the first mile and £1.35 per mile after that.
c Andy is correct. For the length of journey Local Authority A will cost more.

Chapter 14

Exercise 14A

1 a (4, 1)
b (2, −5)
c (−4, −2)
d (4, 3)
e (7.8, −1.2)

f (−5, −4)

g (6, 16)

h (2, 3)

2 a $x = -1,\ y = 4$

b $x = 3,\ y = 0$

c $x = -2,\ y = 3$

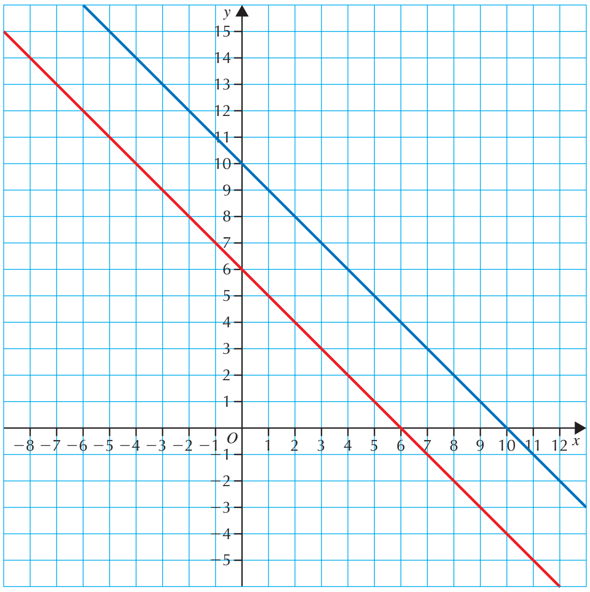
d $x = 2,\ y = 6$

e $x = 1,\ y = 4$

f $x = 4,\ y = -2$

Activity p. 121

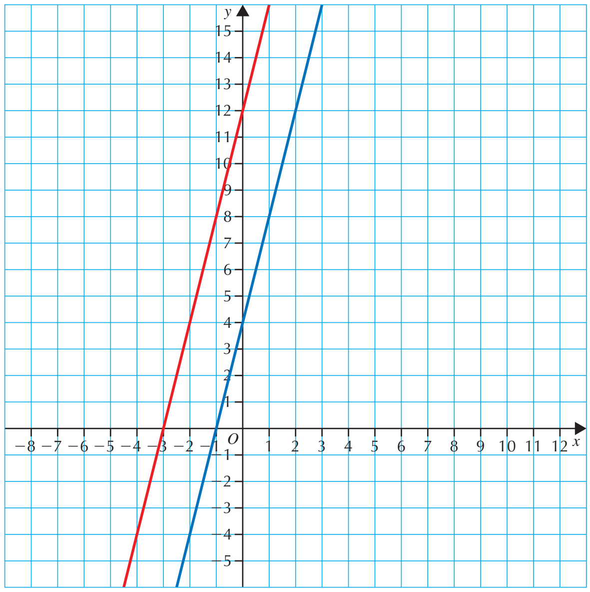
1 a



b The gradient of each line is –1.

c The system of equations has no solutions, because the two lines are parallel.

2 a



b The gradient of each line is 4.

c The system of equation has no solutions, because the two lines are parallel.

3 a The system has no solution, because the two lines are parallel.

b The system has no solution, because the two lines are parallel.

c The system has solution, because gradients are different.

d The system has solution, because gradients are different.

Exercise 14B

1 a $x = -2,\ y = -3$

b $x = 5,\ y = 7$

c $x = 5,\ y = 16$

d $x = -1,\ y = -5$

e $x = 2,\ y = 1$

f $x = -\frac{1}{2},\ y = 6$

g $x = 2,\ y = 1$

h $x = 2,\ y = 1$

2 a $x = 1,\ y = 3$ (1, 3)

b $x = 4,\ y = 2$ (4, 2)

c $x = -2,\ y = -1$ $(-2, -1)$

d $x = -2,\ y = 3$ $(-2, 3)$

e $x = 7, y = -1$ $(7, -1)$

f $x = 4,\ y = -5$ $(4, -5)$

g $x = 3,\ y = -1$ $(3, -1)$

h $x = 5,\ y = 1$ $(5, 1)$

3 a $x = 6,\ y = 2$

b $x = -15,\ y = 4$

c $x = -4,\ y = 7$

d $x = -12,\ y = -10$

e $x = -3,\ y = \frac{5}{2}$

f $x = 5,\ y = 1$

Exercise 14C

1 a $x = 6,\ y = 4$

b $x = 4,\ y = -2$

c $x = -1,\ y = 2$

d $x = -10,\ y = \frac{21}{5}$

e $x = -2,\ y = -4$

f $x = 4,\ y = -1$

g $x = \frac{21}{5},\ y = \frac{8}{5}$

h $x = 3,\ y = -8$

i $x = 4,\ y = -3$

Exercise 14D

1 a $x = 3,\ y = 1$

b $x = 5, y = -3$

c $x = -\frac{29}{7}, y = -\frac{27}{7}$

d $x = 6, y = -2$

e $a = -1, b = -2$

f $p = 3, q = -5$

g $s = -\frac{52}{7}, t = -\frac{18}{7}$

h $c = -\frac{1}{2}, d = \frac{3}{2}$

2 a $x = 2, y = -1$

b $x = -3, y = 1$

c $x = -2, y = -2$

d $x = 5, y = -2$

e $x = -6, y = 5$

f $x = -2, y = \frac{1}{2}$

g $p = 2, q = -3$

h $f = -3, g = -4$

3 The solution obtained by Sally is valid for the second equation only, but not for both equations simultaneously.

Exercise 14E

1 a $5a + 3b = 1.78$

b $2a + b = 0.64$

c i: one apple costs 14p, one banana costs 36p.

2 a $6c + 5z = 89$

b $8c + 3z = 93$

c i: $c = 9$

ii: $z = 7$

3 a $66p + q = 108$

b $45p + q = 74.40$

c £195 will be enough to cover the cost, because the taxi fare for 120 miles is £194.40.

4 $a = 2, b = -3$, so $f(5) = 35$

5 tomato = 32p and onion = 25p

$9t + 40o$ costs £3.88 so Alex has enough money.

6 Each of them sent 50 picture messages.

Chapter 15

Exercise 15A

1 a $x = p - 2$

b $x = w - t$

c $x = q + 5$

d $x = a - 10$

e $x = f - 4$

f $x = mn - k$

2 a $x = \frac{y}{2}$

b $x = -\frac{y}{4}$

c $x = \frac{y-2}{3}$

d $x = \frac{4-y}{5}$

e $x = \frac{y-3}{7}$

f $x = \frac{1-y}{5}$

g $x = \frac{c-a}{b}$

h $x = \frac{1-m}{pqr}$

i $x = \frac{7p-m}{3n}$

3 **a** $t = \frac{d}{v}$

b $m = \frac{w}{g}$

c $R = \frac{V}{I}$

d $b = \frac{V}{lh}$

e $a = \frac{F}{m}$

f $m = \frac{E}{gh}$

g $I = \frac{Q}{t}$

h $h = \frac{l}{Nf}$

i $t = \frac{v-u}{a}$

Exercise 15B

1 **a** $x = 4y$

b $x = 5y$

c $x = \frac{7y}{3}$

d $x = \frac{3y}{2}$

e $x = 6y$

f $x = \frac{5y}{4}$

g $x = 8y - 3$

h $x = 4y - 1$

i $x = 2y + 3$

j $x = \frac{4y-5}{3}$

k $x = \frac{3y+1}{5}$

l $x = \frac{2-5y}{3}$

2 **a** $x = 2y - 12$

b $x = 3y - 3$

c $x = 7y + 14$

d $x = 6y + 24$

e $x = -\frac{5y}{4}$

f $x = 3y - 2$

g $x = y - 5\frac{1}{2}$

h $x = 2\frac{1}{4} - \frac{y}{2}$

3 **a** $W = mg$

b $V = IR$

c $F = ma$

d $E = mgh$

e $v = at + u$

f $u = v - at$

4 **a** $H = 187$ beats per minute

b $A = \frac{2080-10H}{7}$

c Anna is cycling at the maximum heart rate for a person her age and will be at risk if she maintains this heart rate for a long time.

5 **a** Yes. The formula predicts a volume of 4.7 litres or 8.4 pints for a weight of 70 kg.

b $W = \frac{1000v-984}{53.7}$

c **i** A volume of 4.5 litres predicts a weight of 70 kg which is correct.

ii Rounding all numbers in the formula for W to one significant figure gives $W = \frac{1000V-100}{50} = 20(V - 1)$.

Exercise 15C

1 **a** $x = \frac{1}{y}$

b $x = \frac{8}{y}$

c $x = \frac{3}{y} - 5$

d $x = \frac{2}{y} + 1$

e $x = \frac{3}{2y} - \frac{5}{2}$

f $x = \frac{1}{6} - \frac{7}{6y}$

g $x = \frac{1}{2y-12}$

h $x = \frac{3}{4y+32}$

i $x = \frac{4}{y-5} - 3$

j $x = q - \frac{p}{y-1}$

k $x = \frac{g}{fh-yh}$

l $x = \frac{\pi}{my+mp} - \frac{n}{m}$

2 **a** $B = \frac{AR}{A-R}$

b $A = \frac{BCR}{BC-CR-BR}$

Exercise 15D

1 **a** $x = \sqrt{y}$

b $x = \frac{\sqrt{y}}{2}$

c $x = \sqrt{\frac{3y}{2}}$

d $x = \sqrt{\frac{5y}{3}}$

e $x = y^2$

f $x = \frac{y^2}{9}$

g $x = 4y^2$

h $x = y^2 - 5$

i $x = \sqrt{\frac{y-2}{5}}$

j $x = \sqrt{6 - y}$

k $x = \sqrt{\frac{5y-20}{3}}$

l $x = \frac{y^2}{4} + 3$

2 a $x = \sqrt[3]{y}$

b $x = \frac{\sqrt[3]{y}}{2}$

c $x = 1\frac{7}{9}y^2 - 1$

d $x = \sqrt[3]{y} - 2$

e $x = \frac{(3y-15)^2}{4}$

f $x = \frac{(2y+2)^2}{25}$

g $x = \frac{(y-1)^2}{36} + 3$

h $x = \frac{1}{y^2}$

i $x = \frac{16}{y^2}$

j $x = \frac{9}{y^2} - 4$

k $x = \frac{25y^2}{4}$

l $x = \frac{49y^2}{16} + 1$

3 a $r = \frac{\sqrt{M+5}}{2}$

b $q = \sqrt[3]{\frac{I+f}{5}}$

c $k = \sqrt{\frac{3}{W+g}}$

d $d = \frac{1}{2\sqrt{v-P}}$

4 a $h = \frac{(P+10)^2}{9}$

b $t = \frac{g^2}{(Q-v)^2}$

c $r = \frac{g(2-L)^2}{\pi^2}$

d $b = \frac{9}{h(V-4s)^2}$

5 a i $h = \frac{V}{\pi r^2}$

ii $r = \sqrt{\frac{V}{\pi h}}$

b i $h = \frac{3V}{\pi r^2}$

ii $r = \sqrt{\frac{3V}{\pi h}}$

c $v = \sqrt{\frac{2E}{m}}$

d $a = \frac{2(s-ut)}{t^2}$

6 a $d = \sqrt{\frac{GMm}{F}}$

b If d is doubled the force of attraction will be 4 times smaller.

7 a $T = \frac{mv^2}{3k}$

b $m = \frac{3kT}{v^2}$

c The velocity of the gas will double if T is multiplied by 4.

8 a 60 miles per hour

b Student's own work

c 85 feet

d $A = \left(\frac{p^2}{20} - p\right) - \left(\frac{q^2}{20} - q\right) = \frac{(p-q)(p+q+20)}{20}$

9 a 1203 points

b 10.83 seconds

10 a i $\frac{d}{a}$

ii $\frac{d}{b}$

b Total time $= \frac{d}{a} + \frac{d}{b}$; total distance $= 2d$

Average speed $v = \frac{2d}{\frac{d}{a}+\frac{d}{b}} = \frac{2ab}{(a+b)}$

c $b = \frac{av}{2a-v}$

Activity p. 144

Student's own investigation.

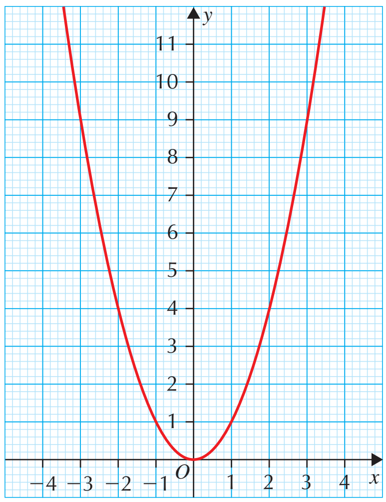
Chapter 16

Activity p. 146

1 a

x	−3	−2	−1	0	1	2	3
x^2	9	4	1	0	1	4	9

b



c The y-axis is an axis of symmetry because any value of x, $y = x^2 = (-x)^2$.

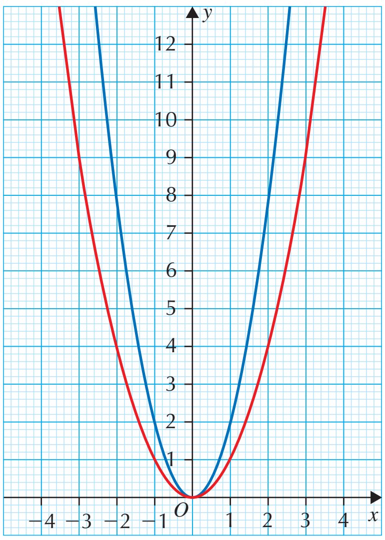
The graph has a minimum turning point at (0, 0) because $x^2 = (-x)^2$ and $y \geq 0$.

The graph never goes below the x-axis because for any value of x, $y \geq 0$.

2 a

x	−3	−2	−1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18

b

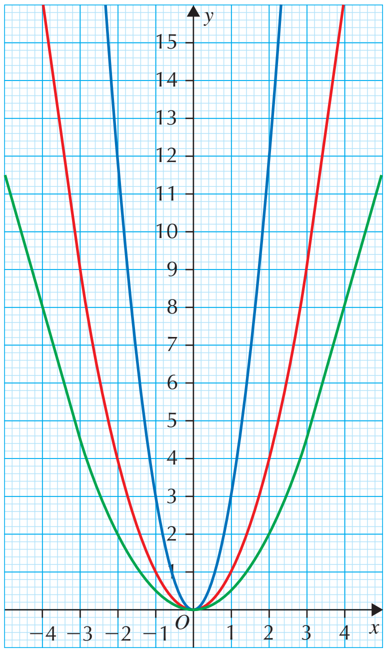


c Only the y-values change on the graph. The turning point and the axis of symmetry do not change.

d The graph of $y = 2x^2$ is narrower, or steeper, than the graph of $y = x^2$.

3

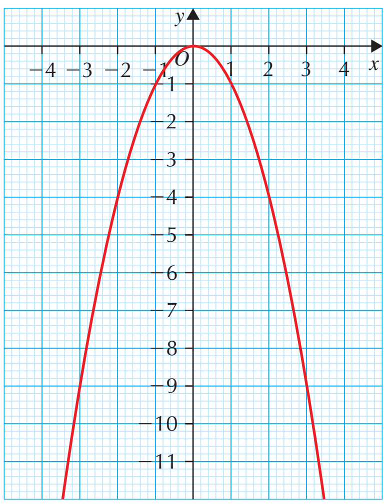
x	−3	−2	−1	0	1	2	3
$3x^2$	27	12	3	0	3	12	27
$\frac{1}{2}x^2$	4.5	2	0.5	0	0.5	2	4.5



4 a

x	−3	−2	−1	0	1	2	3
$-x^2$	−9	−4	−1	0	−1	−4	−9

b



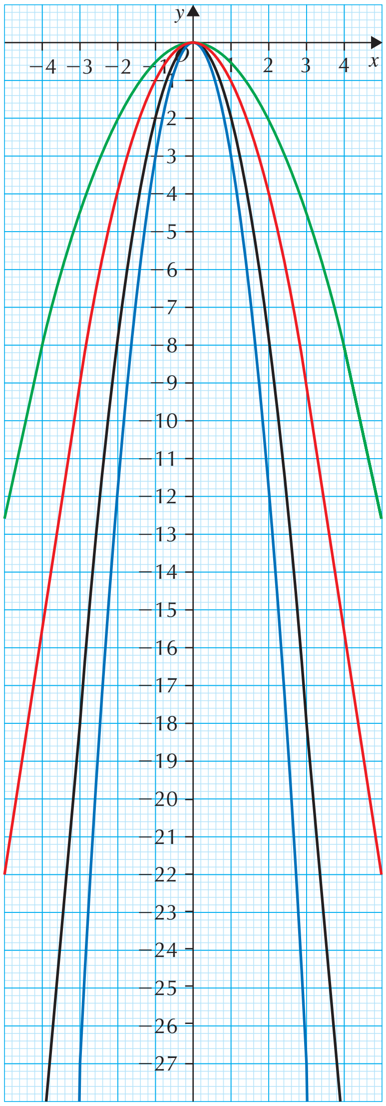
c The y-axis is an axis of symmetry because any value of x, $y = -x^2 = -(-x)^2$.

The graph has a maximum turning point at (0, 0) because $-x^2 = -(-x)^2$ and $y \leq 0$.

The graph never goes above the x-axis because for any value of x, $y \leq 0$.

5

x	−3	−2	−1	0	1	2	3
$-2x^2$	−18	−8	−2	0	−2	−8	−18
$-2x^2$	−27	−12	−3	0	−3	−12	−27
$-\frac{1}{2}x^2$	−4.5	−2	−0.5	0	−0.5	−2	−4.5



Exercise 16A

1 **a** $k = 1 \quad y = x^2$

b $k = 4 \quad y = 4x^2$

c $k = 6 \quad y = 6x^2$

d $k = 20 \quad y = 20x^2$

e $k = \frac{1}{2} \quad y = \frac{1}{2}x^2$

f $k = 8 \quad y = 8x^2$

2 **a** $k = -1 \quad y = -x^2$

b $k = -2 \quad y = -2x^2$

c $k = -3 \quad y = -3x^2$

d $k = -8 \quad y = -8x^2$

e $k = -\frac{5}{16} \quad y = -\frac{5}{16}x^2$

f $k = -\frac{1}{2} \quad y = -\frac{1}{2}x^2$

3 **a** $p = 6$

b $m = 5$

c $k = -3$

4 **a** $k = 2$

b $q = 5$

5 $A_{PQRS} = 125$

6 $A\left(-1, -\frac{1}{6}\right)$, $C\left(1, -\frac{1}{6}\right)$

7 **a** $k = \frac{1}{2}$

b $y = x + 4$

8 $P(p^2, kp^2)$, $Q(q^2, kq^2)$, $R(r^2, kr^2)$, $m_{PQ} = k(p + q)$.

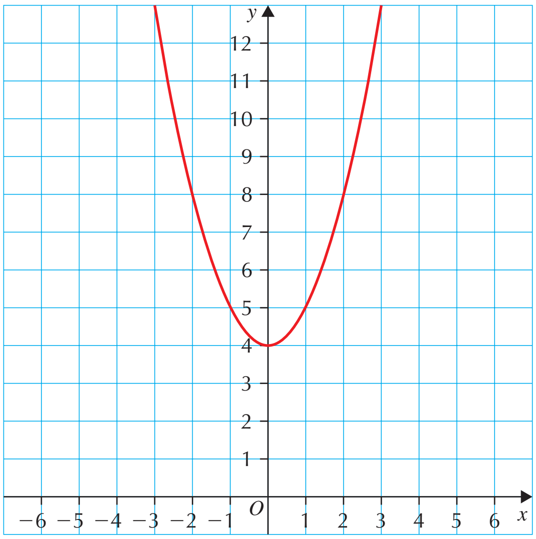
Substitute the coordinates of r into the equation of the straight line to get $r = p + q$.

Activity p. 151

1 **a**

x	–2	–1	0	1	2
$x^2 + 4$	8	5	4	5	8

b



c (0, 4)

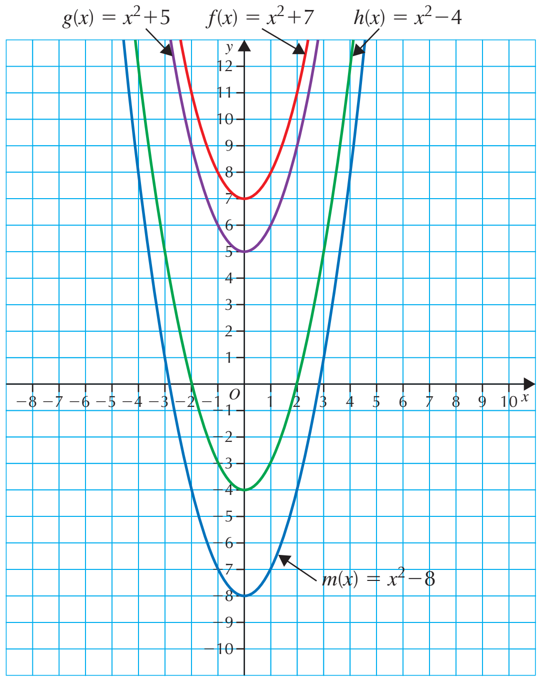
d 4

e $x = 0$

2 **a**

x	–2	–1	0	1	2
$x^2 + 7$	11	8	7	8	11
$x^2 + 5$	9	6	5	6	9
$x^2 – 4$	0	–3	–4	–3	0
$x^2 –8$	–4	–7	–8	–7	–4

b



c **i** (0, 7) **ii** (0, 5) **iii** (0, –4)
iv (0, –8)

d **i** 7 **ii** 5 **iii** –4 **iv** –8.

e The equation of the axis of symmetry is $x = 0$ for all four curves.

3 **a** **i** The minimum value is 1;

ii $x = 0$

b **i** The minimum value is 6;

ii $x = 0$

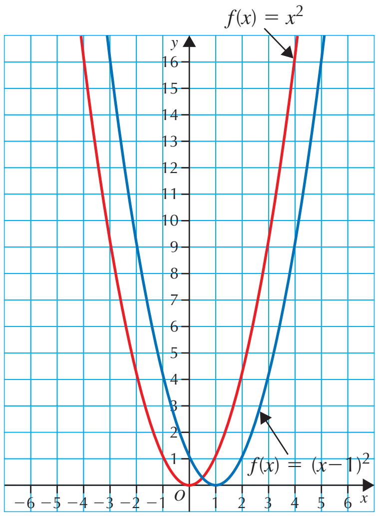
c **i** The minimum value is –3;

ii $x = 0$

d **i** The minimum value is –8;

ii $x = 0$

4 **a, c**



b

x	–1	0	1	2	3
$x - 1$	–2	–1	0	1	2
$(x - 1)^2$	4	1	0	1	4

d **i** The minimum value of the function is 0;

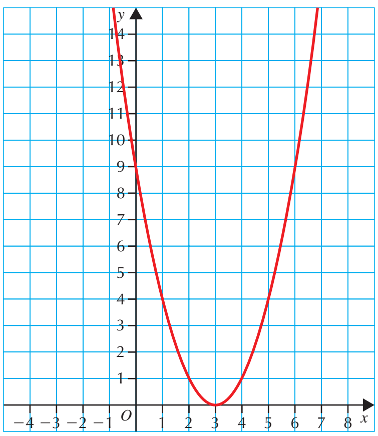
ii $x = 1$; **iii** (1, 0)

e $x = 1$

5 **a**

x	1	2	3	4	5
$x - 3$	–2	–1	0	1	2
$(x - 3)^2$	4	1	0	1	4

b



c **i** The minimum value of the function is 0;

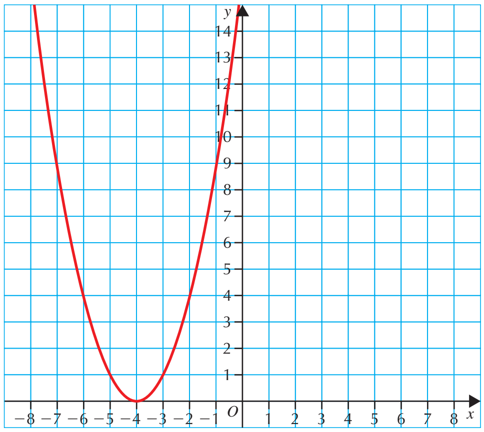
ii $x = 3$; **iii** (3, 0)

d $x = 3$

6 **a**

x	–4	–3	–2	–1	0
$x + 2$	–2	–1	0	1	2
$(x + 2)^2$	4	1	0	1	4

b



c **i** The minimum value of the function is 0;

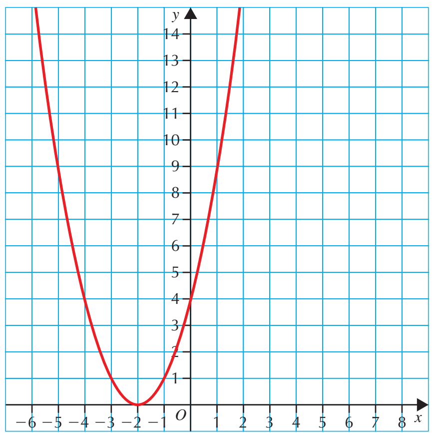
ii $x = -2$; **iii** (–2, 0)

d $x = -2$

7 **a**

x	–6	–5	–4	–3	–2
$x + 4$	–2	–1	0	1	2
$(x + 4)^2$	4	1	0	1	4

b



c **i** The minimum value of the function is 0;

ii $x = -4$; **iii** (–4, 0)

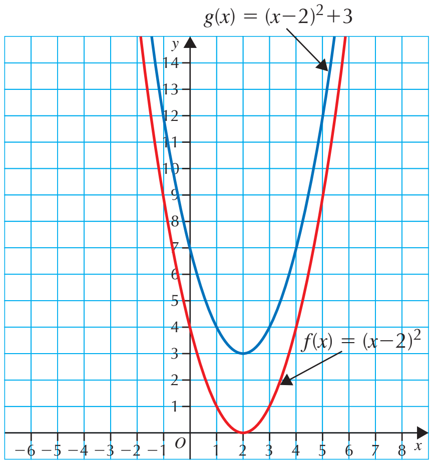
d $x = -4$

8 **a** **i** The minimum value is 0;
ii $x = 2$; **iii** (2, 0); **iv** $x = 2$
b **i** The minimum value is 0;
ii $x = 5$; **iii** (5, 0); **iv** $x = 5$
c **i** The minimum value is 0;
ii $x = -1$; **iii** (−1, 0); **iv** $x = -1$
d **i** The minimum value is 0;
ii $x = -6$; **iii** (−6, 0); **iv** $x = -6$
e **i** The minimum value is 0;
ii $x = -a$; **iii** (−a, 0); **iv** $x = -a$
f **i** The minimum value is 0;
ii $x = a$; **iii** (a, 0); **iv** $x = a$

9 **a** $(x - 2)^2 + 3 = 7, 4, 3, 4, 7$

x	0	1	2	3	4
$(x-2)^2$	4	1	0	1	4
$(x-2)^2+3$	7	4	3	4	7

b

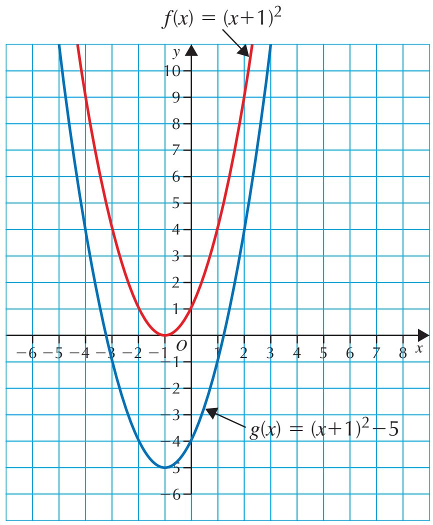


c **i** The minimum value is 3;
ii $x = 2$; **iii** (2, 3)

10 **a**

x	−3	−2	−1	0	1
$(x+1)^2$	4	1	0	1	4
$(x+1)^2-5$	−1	−4	−5	−4	−1

b



c **i** The minimum value is −5;
ii $x = -1$; **iii** (−1, −5)

11 **a** **i** The minimum value is 3;
ii $x = 4$; **iii** (4, 3)
b **i** The minimum value is 1;
ii $x = 8$; **iii** (8, −1)
c **i** The minimum value is 10;
ii $x = -3$; **iii** (−3, 10)
d **i** The minimum value is −4;
ii $x = -6$; **iii** (−6, −4)

12 **a** **i** The maximum value is 1;
ii $x = 0$; **iii** (0, 1)
b **i** The maximum value is 6;
ii $x = 0$; **iii** (0, 6)
c **i** The maximum value is −3;
ii $x = 0$; **iii** (0, −3)
d **i** The maximum value is −8;
ii $x = 0$; **iii** (0, −8)

Exercise 16B

1 **a** $p = -3, q = 1$
b $p = -1, q = 5$
c $p = -4, q = -2$
d $p = -2, q = 4$

e $p = 2, q = 3$
f $p = 5, q = 4$

2 a $p = -5, q = -4$
b $p = 2, q = 1$
c $p = -3, q = -6$
d $p = 3, q = -5$
e $p = -7, q = -4$
f $p = 6, q = 5$

3 a $x = 4$
b B (1, 11)

4 $p = 3, q = -4$

5 $p = 2, q = 17$

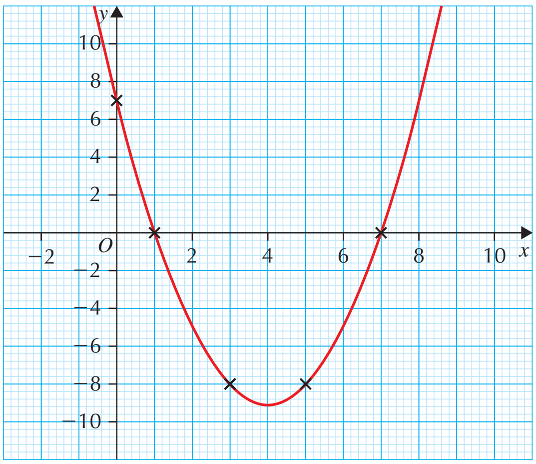
Chapter 17

Activity p. 159

1 a

x	0	1	3	5	7	9
$(x - 1)$	–1	0	2	4	6	8
$(x - 7)$	–7	–6	–4	–2	0	2
$(x - 1)(x - 7)$	7	0	–8	–8	0	16

b

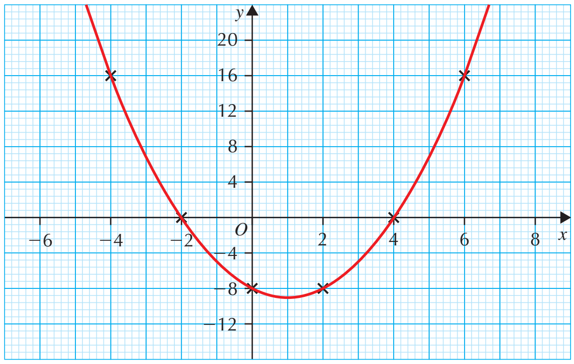


c $x = 1, x = 7$
d (4, –9)

2 a

x	–4	–2	0	2	4	6
$(x + 2)$	–2	0	2	4	6	8
$(x - 4)$	–8	–6	–4	–2	0	2
$(x + 2)(x - 4)$	16	0	–8	–8	0	16

b

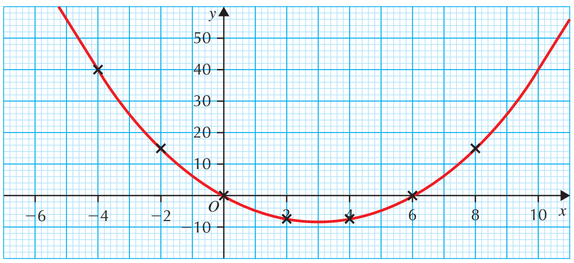


c $x = -2, x = 4$
d (1, –9)

3 a

x	–4	–2	0	2	4	6	8
$(x - 6)$	–10	–8	–6	–4	–2	0	2
$x(x - 6)$	40	16	0	–8	–8	0	16

b



c $x = 0, x = 6$
d (3, –9)

4 a i $x = 2, x = 6$
ii $x = 4$
iii –4

b **i** $x = 3, x = 5$
ii $x = 4$
iii -1

c **i** $x = -1, x = 3$
ii $x = 1$
iii -4

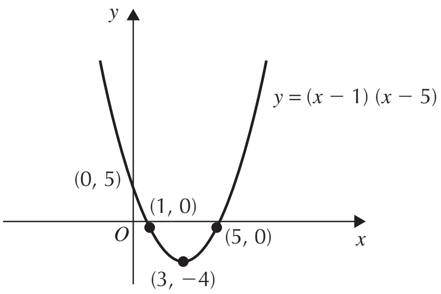
d **i** $x = 0, x = -4$
ii $x = 2$
iii 12

e **i** $x = -1, x = -7$
ii $x = -4$
iii -9

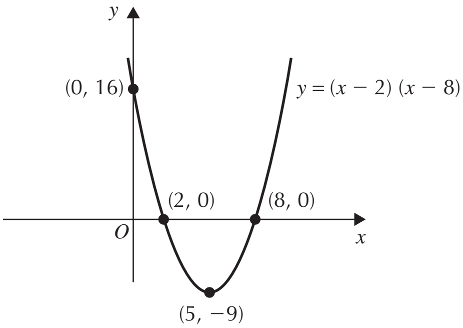
f **i** $x = -5, x = 5$
ii $x = 0$
iii -25

Exercise 17A

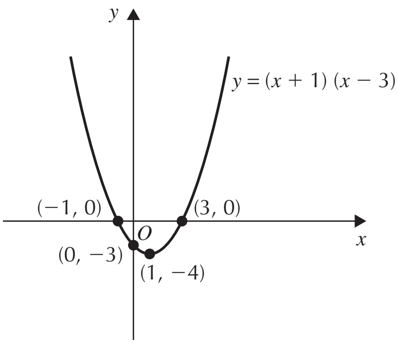
1 **a**



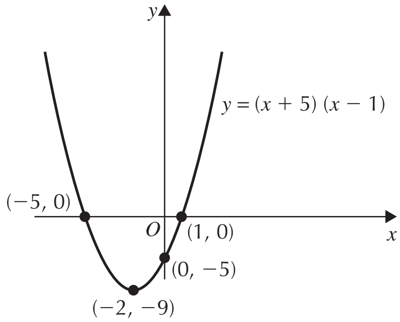
b



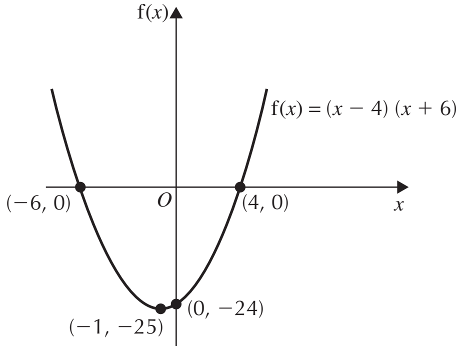
c



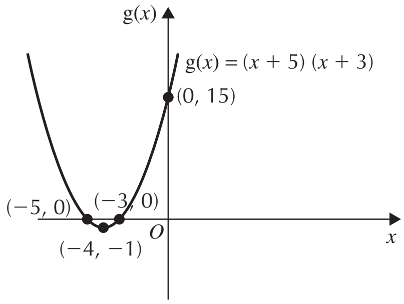
d



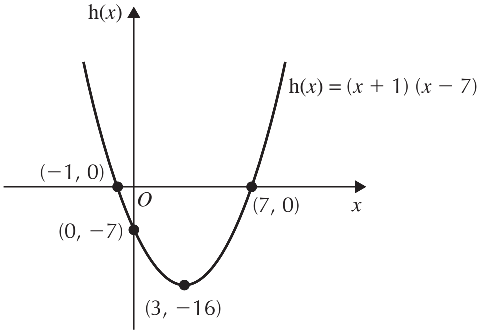
e



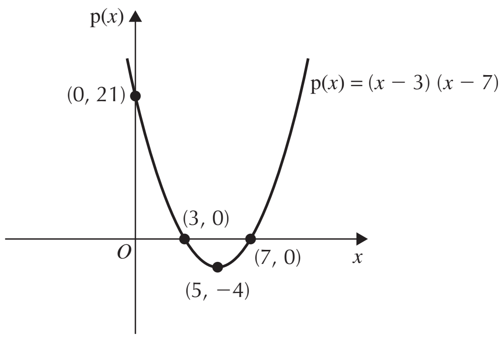
f



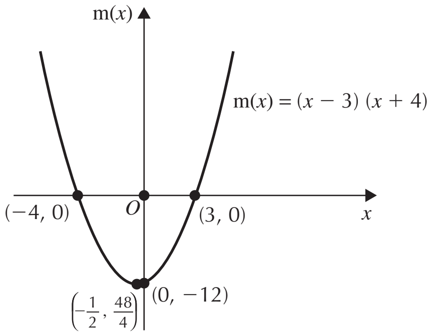
g



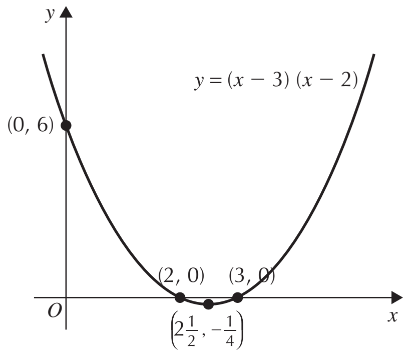
h



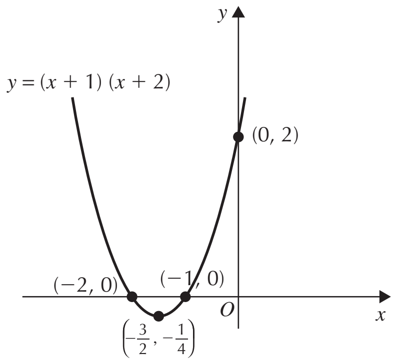
i



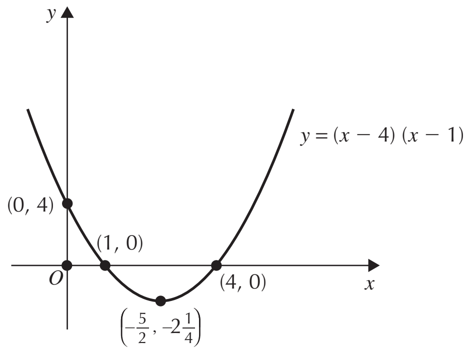
2 a



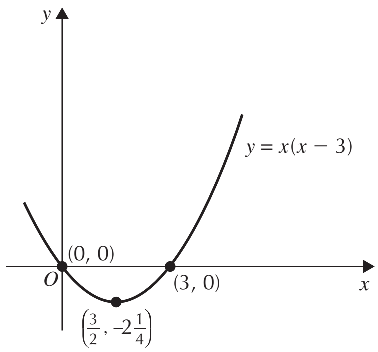
b



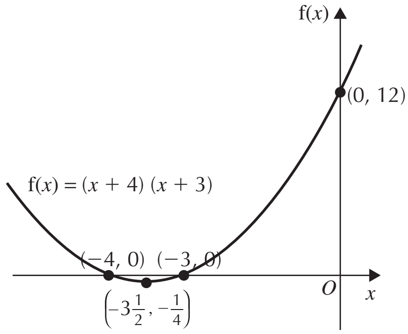
c



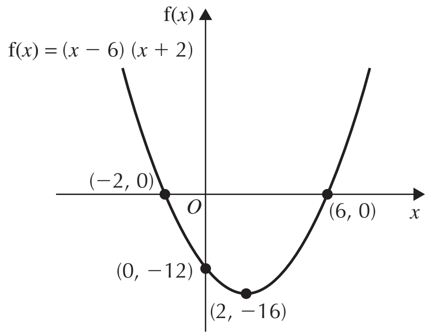
d



e



f



3 a $m = 3, n = 5$ or $m = 5, n = 3$

b $m = 1, n = 5$ or $m = 5, n = 1$

c $m = -2, n = -4$ or $m = -4, n = -2$

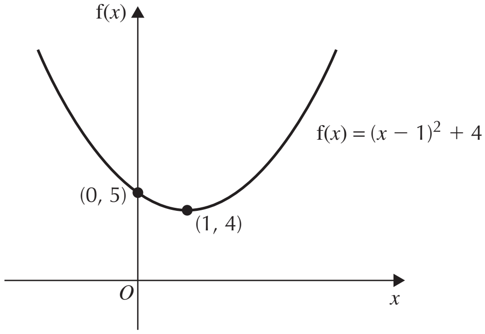
d $m = -3, n = 1$ or $m = 1, n = -3$

e $m = -6, n = 0$ or $m = 0, n = -6$

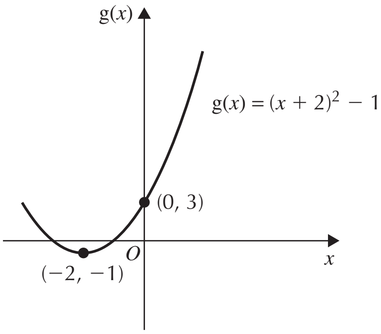
f $m = -3, n = 2$ or $m = 2, n = -3$

Exercise 17B

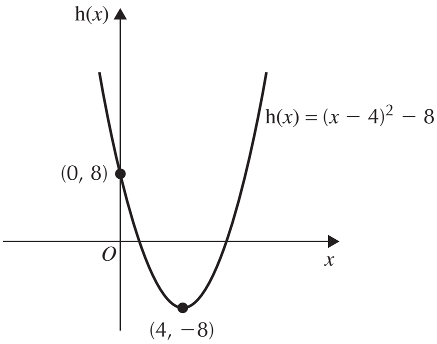
1 **a**



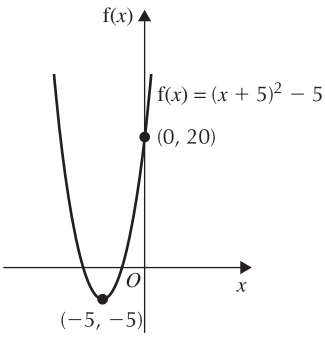
b



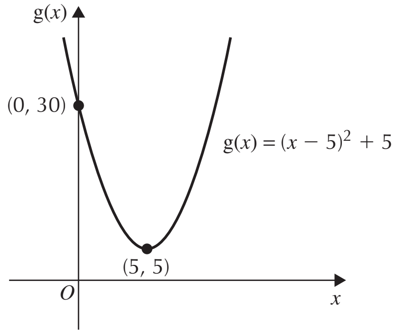
c



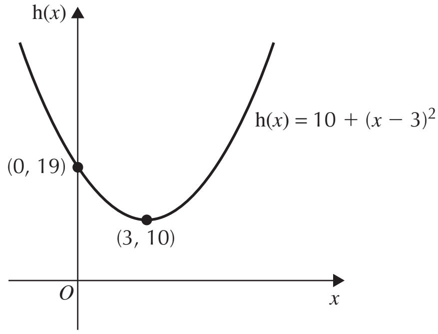
d



e



f



2 **a** Since the square of a real number is always positive, $y = x^2$ will have a turning point at $x = 0$ and must have a minimum value of $y = 0$.

b Since $y = -x^2$ is always negative, the function must have a *maximum* value of 0.

3 **a** $y = -x^2 + 10$ is the function $y = -x^2$ moved 10 units up the *y-axis*. The maximum value of $y = -x^2 + 10$ is $0 + 10 = 10$.

b $-x^2 + 10 = 10 - x^2$ and so the maximum value of the function is 10.

4 **a** $y = (x - 3)^2$ is the function $y = x^2$ moved 3 units along *x*-axis. As $y = x^2$ has a minimum value of 0, the minimum value of $y = (x - 3)^2$ is also 0.

b The function $y = -(x - 3)^2$ is a reflection of $y = (x - 3)^2$ across the *x*-axis. Since the function is always negative it must have a maximum value of 0.

5 **a** Minimum value = 8

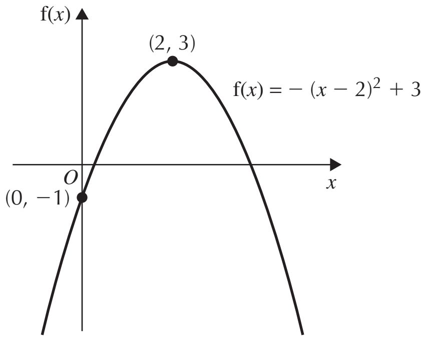
b The function $y = -(x + 2)^2 + 8$ is a reflection of $y = (x + 2)^2 + 8$ across the *x*-axis and will have a maximum value of 8.

c Kayleigh is right. Addition and subtraction rank equally in the order of operations and so the function can be written in either form.

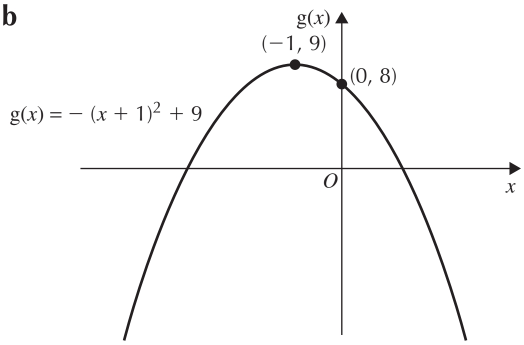
6 The function will be reflected across the x-axis and will have maximum value of q when $x = -p2$.

Exercise 17C

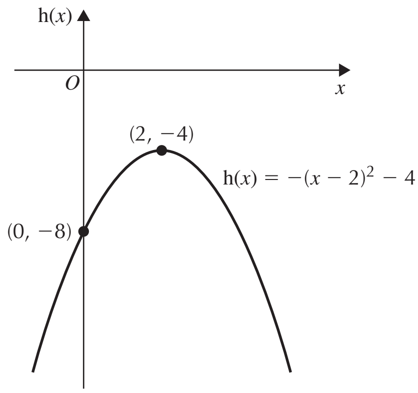
1 a



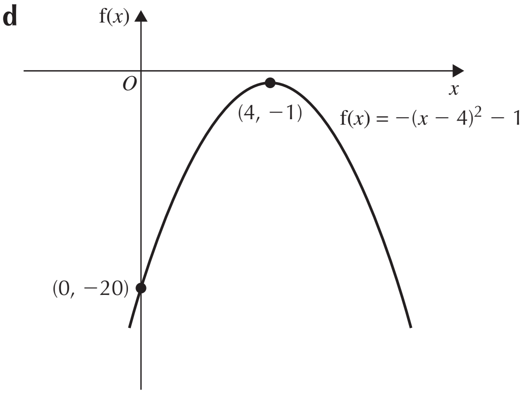
b



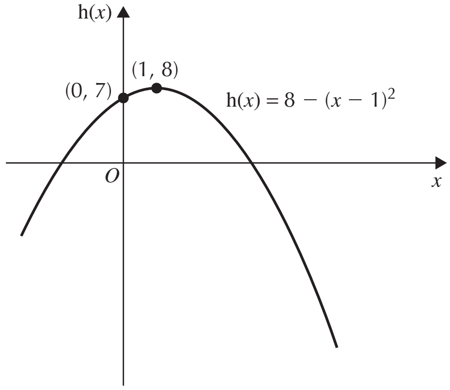
c



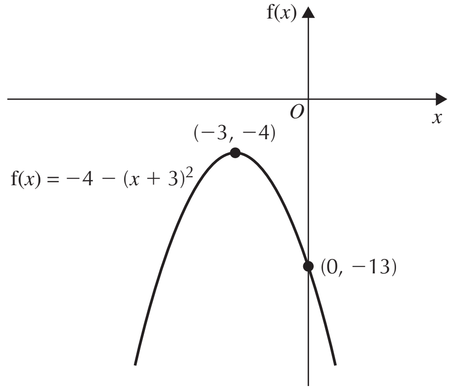
d



e



f



2 a For $k > 0$, $y = kx^2$ is always positive and is smallest when $x = 0$ and so must have a minimum value of 0.

b When $k < 0$, the function is always negative. It will be largest when $x = 0$ resulting in a maximum value of 0.

3 a The minimum value of $y = (x - 2)^2$ is 0.

b The minimum value of $y = 3(x - 2)^2$ is 0.

c When $k > 0$, the minimum value of $y = k(x - 2)$ is 0.

d The minimum values are the same.

4 **a** The turning point is at (–2, –5).

b Both functions will have the same turning point as they are both smallest when $x = -2$. At the turning point both function have a value $y = 5$.

c The coordinates of the turning point would always be the same as they are both always smallest at $x = -2$ when $y = 5$.

5 **a** The maximum value is 15.

b **i** 15 **ii** 15

c Both functions have the same turning point.

6 **a** $(-p, q)$

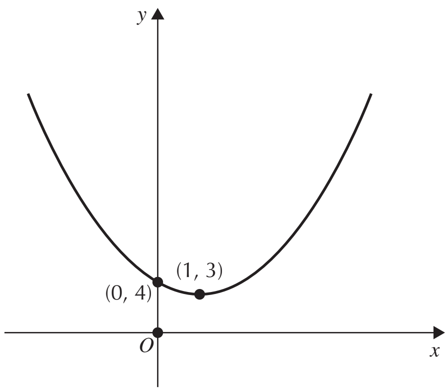
b $(-p, q)$

Exercise 17D

1 **a** **i** $b^2 - 4ac = -12$

ii (0, 4)

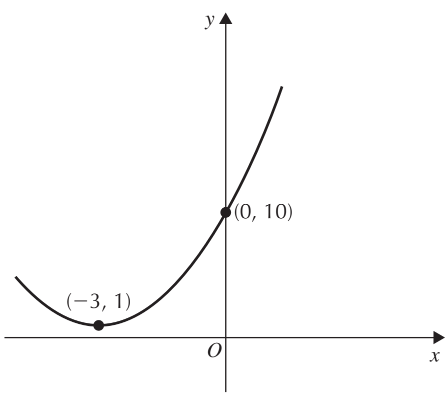
iii (1, 3)



b **i** $b^2 - 4ac = -4$

ii (0, 10)

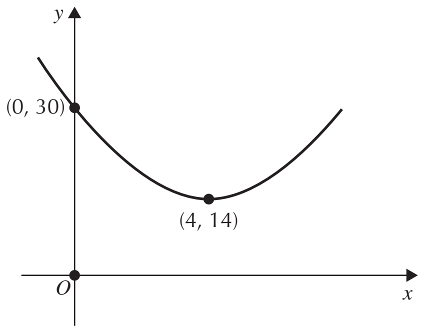
iii (–3, 1)



c **i** $b^2 - 4ac = -56$

ii (0, 30)

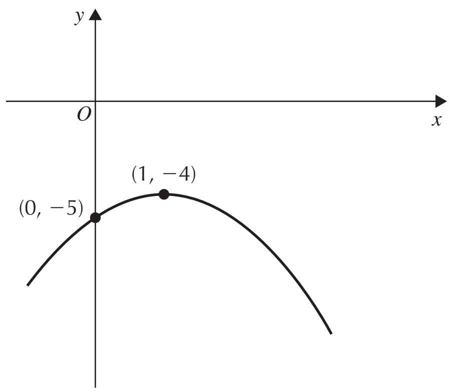
iii (4, 14)



d **i** $b^2 - 4ac = -16$

ii (0, –5)

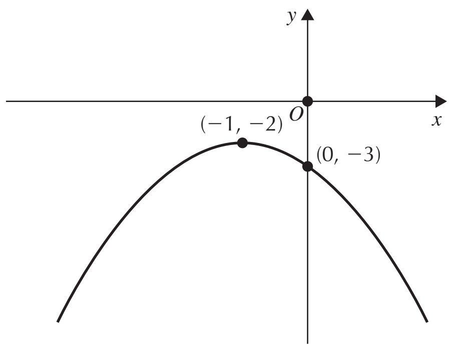
iii (1, –4)



e **i** $b^2 - 4ac = -8$

ii (0, –3)

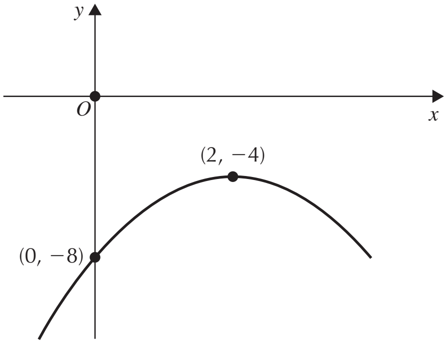
iii (–1, –2)



f **i** $b^2 - 4ac = -16$

ii $(0, -8)$

iii $(2, 4)$



2 **a**

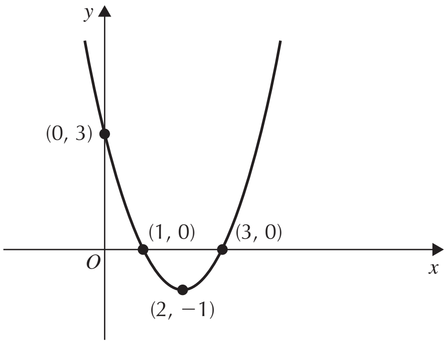
i	$(x-1)^2 + 3$	**ii**	3
i	$(x+3)^2 + 1$	**ii**	1
i	$(x-4)^2 + 14$	**ii**	14
i	$-(x-1)^2 - 4$	**ii**	-4
i	$-(x+1)^2 - 2$	**ii**	-2
i	$-(x-2)^2 - 4$	**ii**	-4

b Minimum values for the functions for which $k > 0$ are always above the x-axis. Maximum values for the functions for which $k < 0$ are always below the x-axis.

3 **a** **i** $b^2 - 4ac = 4 = 2^2$

ii $(3, 0), (1, 0), (0, 3)$

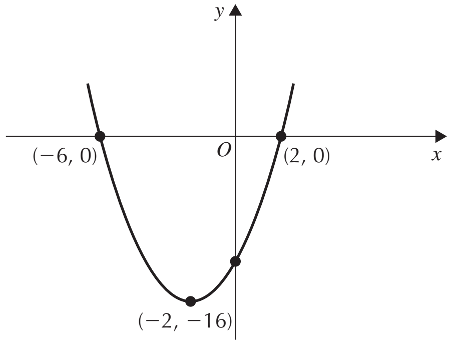
iii $(2, -1)$



b **i** $b^2 - 4ac = 64 = 8^2$

ii $(-6, 0), (2, 0), (0, -12)$

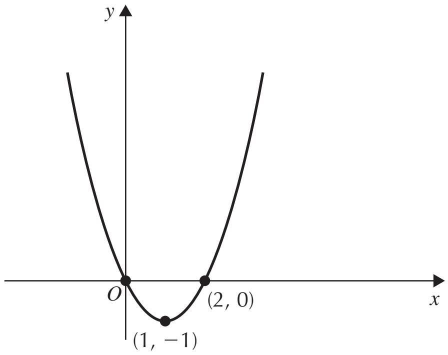
iii $(-2, -16)$



c **i** $b^2 - 4ac = 4 = 2^2$

ii $(0, 0), (2, 0), (0, 0)$

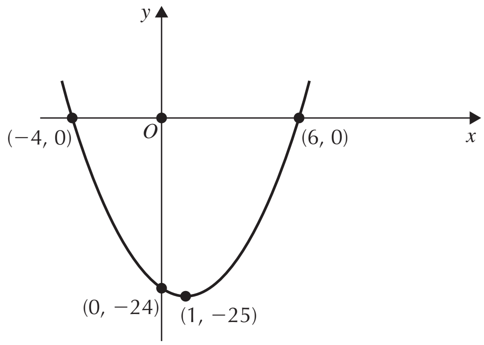
iii $(1, -1)$



d **i** $b^2 - 4ac = 100 = 10^2$

ii $(6, 0), (-4, 0), (0, -24)$

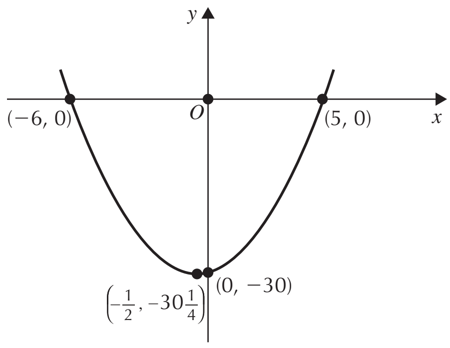
iii $(1, -25)$



e **i** $b^2 - 4ac = 121 = 11^2$

ii $(5, 0), (-6, 0), (0, -30)$

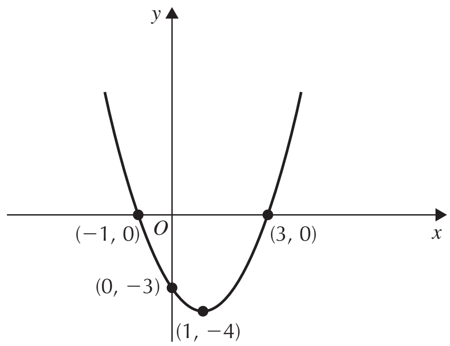
iii $\left(-\frac{1}{2}, 30\frac{1}{4}\right)$



f **i** $b^2 - 4ac = 16 = 4^2$

ii (3, 0), (–1, 0), (0, –3)

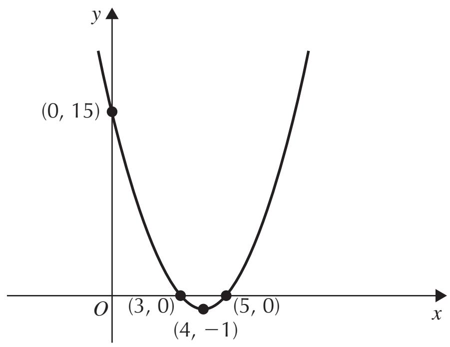
iii (1, –4)



g **i** $b^2 - 4ac = 4 = 2^2$

ii (5, 0), (3, 0), (0, 15)

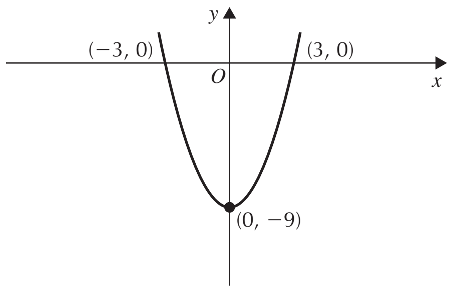
iii (4, –1)



h **i** $b^2 - 4ac = 36 = 6^2$

ii (3, 0), (–3, 0), (0,–9)

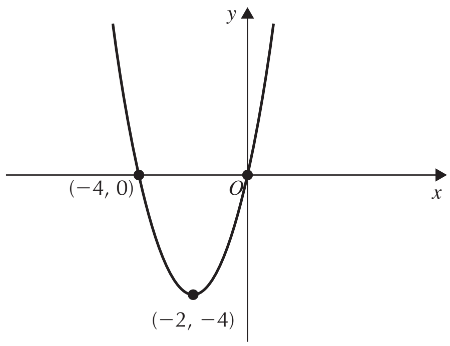
iii (0, –9)



i **i** $b^2 - 4ac = 16 = 4^2$

ii (–4, 0), (0, 0), (0, 0)

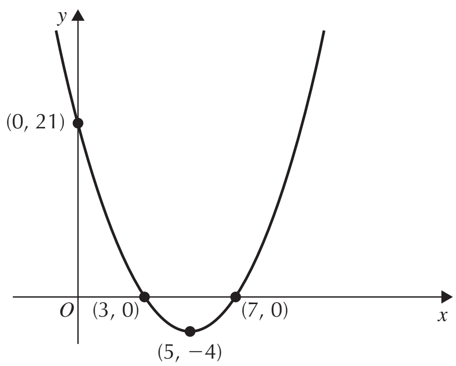
iii (–2, –4)



j **i** $b^2 - 4ac = 16 = 4^2$

ii (7, 0), (3, 0), (0, 21)

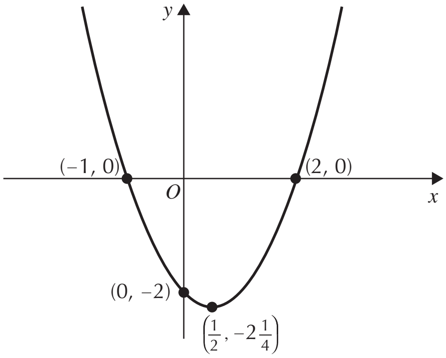
iii (5, –4)



k **i** $b^2 - 4\text{ac} = 9 = 3^2$

ii (2, 0), (–1, 0), (0, –2)

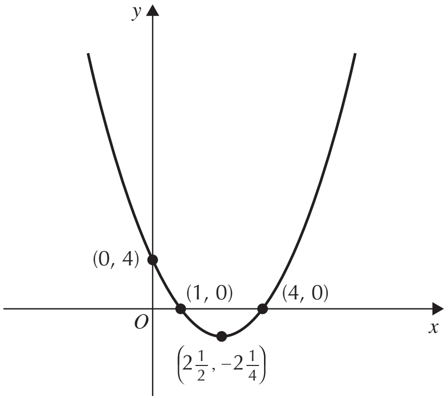
iii $\left(\frac{1}{2}, -2\frac{1}{4}\right)$



l **i** $b^2 - 4ac = 9 = 3^2$

ii (4, 0), (1, 0), (0, 4)

iii $\left(2\frac{1}{2}, -2\frac{1}{4}\right)$



4 **a** **i** $b^2 - 4ac = 0$

ii one real root

iii Graph only touches the x-axis at one point

b **i** $b^2 - 4ac = 0$

ii one real root

iii Graph only touches the x-axis at one point

c **i** $b^2 - 4ac = 0$

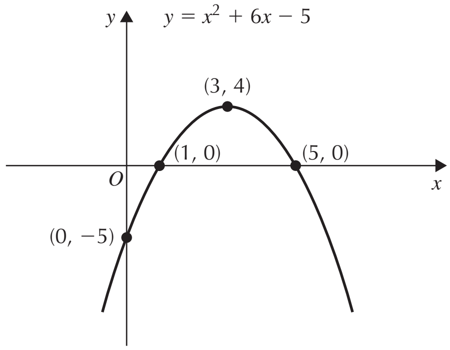
ii one real root

iii Graph only touches the x-axis at one point

5 **a** **i** $b^2 - 4ac = 16$

ii (1, 0), (5, 0), (0, –5)

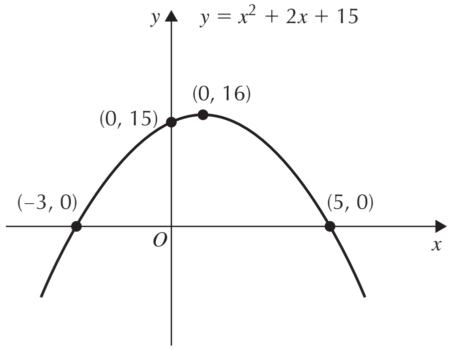
iii (3, 4)



b **i** $b^2 - 4ac = 64$

ii (–3, 0), (5, 0), (0, 15)

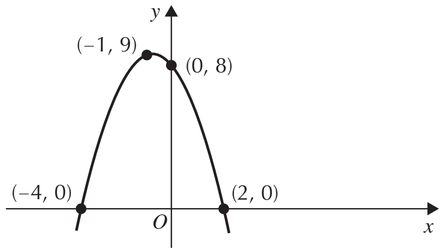
iii (1, 16)



c **i** $b^2 - 4ac = 36$

ii (–4, 0), (2, 0), (0, 8)

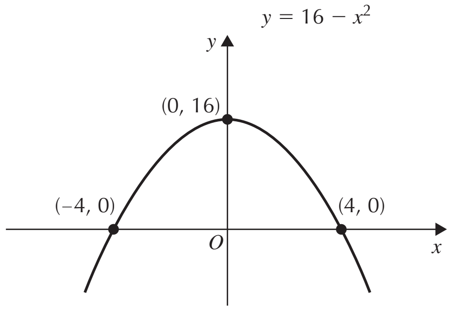
iii (–1, 9)



d **i** $b^2 - 4ac = 64$

ii (4, 0), (–4, 0), (0, 16)

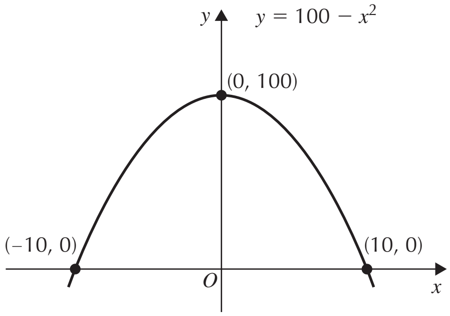
iii (0, 16)



e **i** $b^2 - 4ac = 400$

ii (10, 0), (−10, 0), (0, 100)

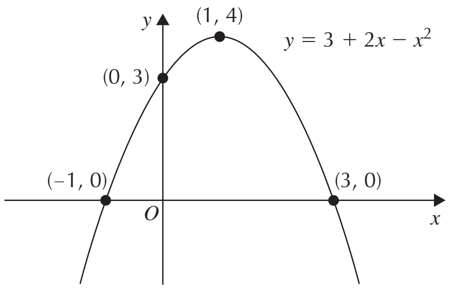
iii (0, 100)



f **i** $b^2 - 4ac = 16$

ii (3, 0), (−1, 0), (0, 3)

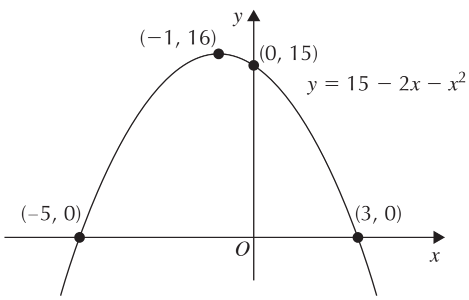
iii (1, 4)



g **i** $b^2 - 4ac = 64$

ii (3, 0), (−5, 0), (0, 15)

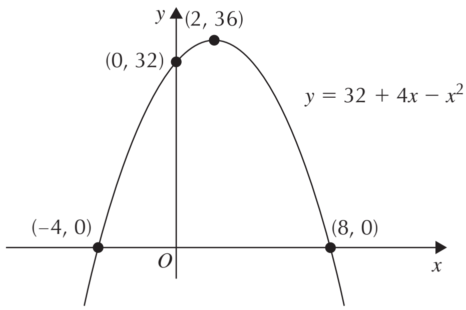
iii (−1, 16)



h **i** $b^2 - 4ac = 144$

ii (8, 0), (−4, 0), (0, 32)

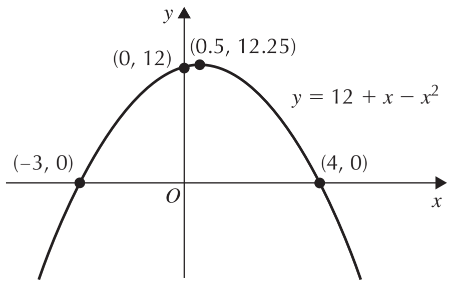
iii (2, 36)



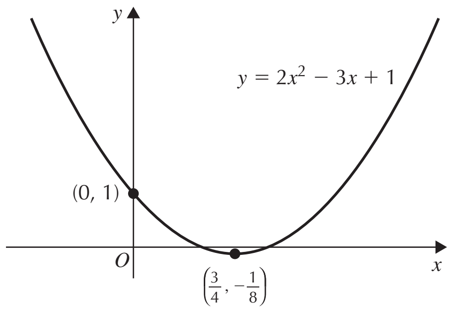
i **i** $b^2 - 4ac = 49$

ii (4, 0), (−3, 0), (0, 12)

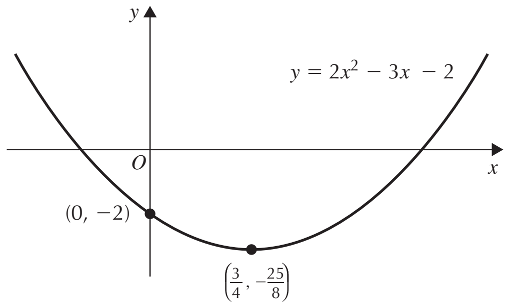
iii $\left(\frac{1}{2}, 12\frac{1}{4}\right)$



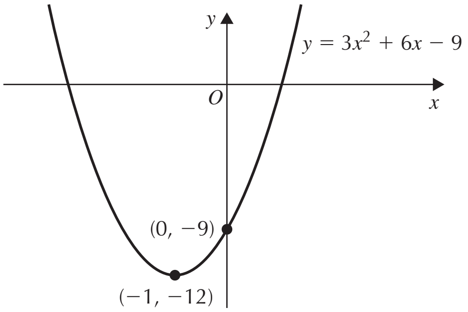
6 **a**



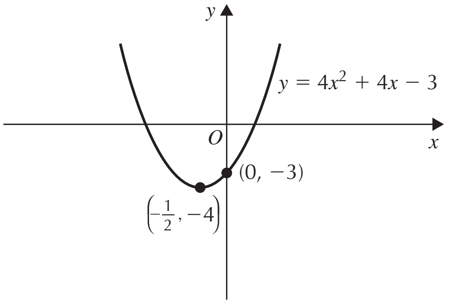
b



c



d



Activity p. 173

a Pupil's own answer

b **i** $\frac{c}{a} - \frac{b^2}{4a^2} + t^2$

ii $f\left(-\frac{b}{2a} + t\right) = -\frac{b^2}{4a^2} + t^2 + \frac{c}{a}$

$f\left(-\frac{b}{2a} - t\right) = -\frac{b^2}{4a^2} + t^2 + \frac{c}{a}$

iii This shows the minimum value of the function is at $x = -\frac{b}{2a}$.

c **i** $c - \frac{b^2}{4a} + at^2$

ii Proven in same manner as in part **bii.**

d Error in question. Correct question is:

Show that $a(x - m)(x - n)$ can be written as $ax^2 - (m + n)\ ax + amn$.

This is solved by expanding the brackets.

e m, n

f Student's own answer.

Exercise 17E

1

Function	General form	Completed square form	Root form
a Function 1	$x^2 - 4x + 3$	$(x - 2)^2 - 1$	$(x - 1)(x - 3)$
b Function 2	$x^2 + 4x + 3$	$(x + 2)^2 - 1$	$(x + 3)(x + 1)$
c Function 3	$x^2 - 2x - 15$	$(x - 1)^2 - 16$	$(x + 3)(x - 5)$
d Function 4	$x^2 - 4x - 12$	$(x - 2)^2 - 16$	$(x + 2)(x - 6)$
e Function 5	$x^2 + 10x - 24$	$(x + 5)^2 - 1$	$(x + 4)(x + 6)$
f Function 6	$x^2 + 6x + 5$	$(x + 3)^2 - 4$	$(x + 1)(x + 5)$

2 $q = -31$

3 Answers may vary, two possible answers are:

$b^2 - 4ac = -16$ and so there are no real roots and the graph does not cross or touch the x-axis.

The function has a minimum value of 4 which is above the x-axis.

4 $b = 4, c = -5$

5 $q = -4$

6 $a = 1, b = 7$ or $a = 7, b = 1$

7 5, −11

Chapter 18

Exercise 18A

1 **a** **i** $x = 2$ **ii** (2, 5); minimum

b **i** $x = 4$ **ii** (4, 1); minimum

c **i** $x = 8$ **ii** (8, −3); minimum

d i $x = 6$ ii $(6, -2)$; maximum
e i $x = \frac{1}{2}$ ii $\left(\frac{1}{2}, \frac{3}{4}\right)$; minimum
f i $x = -1$ ii $(-1, 9)$; minimum
g i $x = -7$ ii $(-7, 2)$; maximum
h i $x = -3$ ii $(-3, -5)$; minimum
i i $x = -10$ ii $(-10, -4)$; maximum

2 a $x = 6$
b $a = -6$
c $b = -16$
d $(0, 20)$

3 a $a = 8$
b $S = (3, 0)$, $T = (13, 0)$
c $(8, 25)$
d $(0, -39)$

4 a $B = (22, 0)$
b $S = (10, -144)$
c 1000 units^2

5 a $p = 6$, $q = 21$
b i $M = (0, 9)$ ii $a = 6$, $b = 5$

6 $m = 6$, $n = 5$

Exercise 18B

1 a 11 feet
b 36 feet
c 2.75 seconds

2 a 1940 bacteria
b 0.5° Celsius
c 9° Celsius

3 a 45 metres
b 125 metres
c 9 seconds

4 a The cordoned off area will be a rectangle of length x and width $(20 - x)$. The area is given by $x(20 - x) = 20x - x^2$.
b 100 m^2

5 a £75
b £562,500
c £150

6 a i 71.7°Farenheit ii $P = 92\%$
b 0%
c Between 69.9° Farenheit and 73.5° Farenheit

7 a 57%
b 81

Activity p. 182

a i $V = (4x^3 - 10x^2 + 6x)\text{m}^3$;
ii $SA = (6 - 4x^2)\text{m}^2$
b $P = 1400x^3 - 3410x^2 - 2100x$
c $0 < x < 1$
d $x_{max} = 0.81m$, giving a maximum profit of £208.

Chapter 19

Exercise 19A

1 a $x = 4$ or $x = 2$
b $x = 0$ or $x = -4$
c $x = \frac{3}{2}$ or $x = -2$
d $x = -\frac{3}{2}$
e $x = 0$ or $x = -\frac{7}{2}$
f $x = \frac{1}{2}$ or $x = -7$
g $x = 0$ or $x = \frac{5}{3}$
h $x = -1$ or $x = \frac{2}{3}$
i $x = 0$ or $x = -\frac{4}{3}$

2 a $x(4x - 1) = 0$
$x = 0$ or $x = \frac{1}{4}$
b $3x(2x + 3) = 0$
$x = 0$ or $x = -\frac{3}{2}$
c $5x(3 - 5x) = 0$
$x = 0$ or $x = \frac{3}{5}$
d $2x(2x - 5) = 0$
$x = 0$ or $x = \frac{5}{2}$
e $5x(x - 1) = 0$
$x = 0$ or $x = 1$
f $4x(4 - x) = 0$
$x = 0$ or $x = 4$
g $x(11 + x) = 0$
$x = 0$ or $x = -11$
h $2x(2 - 3x) = 0$
$x = 0$ or $x = \frac{2}{3}$

3 **a** $(2x+3)(2x-3)=0$

$x=-\frac{3}{2}$ or $x=\frac{3}{2}$

b $(5p-4)(5p+4)=0$

$p=\frac{4}{5}$ or $p=-\frac{4}{5}$

c $(2-m)(2+m)=0$

$m=2$ or $m=-2$

d $(x-9)(x+9)=0$

$x=9$ or $x=-9$

e $(x-7)(x+7)=0$

$x=7$ or $x=-7$

f $(3x-10)(3x+10)=0$

$x=\frac{10}{3}$ or $x=-\frac{10}{3}$

g $(11-9q)(11+9q)=0$

$q=\frac{11}{9}$ or $q=-\frac{11}{9}$

h $(8-2t)(8+2t)=0$

$t=4$ or $t=-4$

4 **a** $(x+5)(x+3)=0$

$x=-5$ or $x=-3$

b $(t-3)(t-1)=0$

$t=3$ or $t=1$

c $(x-5)(x+2)=0$

$x=5$ or $x=-2$

d $(x-3)(x-2)=0$

$x=3$ or $x=2$

e $(x-10)(x+2)=0$

$x=10$ or $x=-2$

f $(z+9)(z+5)=0$

$z=-9$ or $z=-5$

g $(y+6)(y-2)=0$

$y=-6$ or $y=2$

h $(w+3)(w-2)=0$

$w=-3$ or $w=2$

i $(r+7)(r-2)=0$

$r=-7$ or $r=2$

5 **a** $(2r+1)(r+1)=0$

$r=-\frac{1}{2}$ or $r=-1$

b $-(t-3)(t-4)=0$

$t=3$ or $t=4$

c $(3s+2)(s-2)=0$

$s=-\frac{2}{3}$ or $s=2$

d $-(p+3)(2p+1)=0$

$p=-3$ or $p=-\frac{1}{2}$

e $(3w-4)(w+3)=0$

$w=\frac{4}{3}$ or $w=-3$

f $-(x-5)(6x-1)=0$

$x=5$ or $x=\frac{1}{6}$

g $-(12x^2-24x-12)=0$

$-(x-1)(x-1)=0$

$x=1$

h $(2m-3)(m+5)=0$

$m=\frac{3}{2}$ or $m=-5$

i $(p-1)(5p+18)=0$

$p=1$ or $p=-\frac{18}{5}$

6 **a** $p(p+4)=0$

$p=0$ or $p=-4$

b $(x+7)(x+7)=0$

$x=-7$

c $(x+1)(2x-5)=0$

$x=-1$ or $x=\frac{5}{2}$

d $(6+p)(6-p)=0$

$p=-6$ or $p=6$

e $6m(2m+3)=0$

$m=0$ or $m=-\frac{3}{2}$

f $-(x+7)(5x+3)=0$

$x=-7$ or $x=-\frac{3}{5}$

g $2(2x-5)(2x+5)=0$

$x=\frac{5}{2}$ or $x=-\frac{5}{2}$

h $-2(x-5)(3x+4)=0$

$x=5$ or $x=-\frac{4}{3}$

i $-2(4m-7)(4m+7)=0$

$m=\frac{7}{4}$ or $m=-\frac{7}{4}$

j $3(a-5)(2a-1)$

$a=5$ or $a=\frac{1}{2}$

k $3(2x-5)(2x+5)$

$x=\frac{5}{2}$ or $x=-\frac{5}{2}$

l $5(x+3)(x+4)$

$x=-3$ or $x=-4$

Exercise 19B

1 **a** $4x(x-2)=0$

$x=0$ or $x=2$

b $(2x+3)(2x-3)=0$

$x=-\frac{3}{2}$ or $x=\frac{3}{2}$

c $(x-1)(x-2)=0$

$x=1$ or $x=2$

d $(2x-1)(x+3)=0$

$x=\frac{1}{2}$ or $x=-3$

e $(x-3)(x-3)=0$

$x=3$

f $(x+9)(x-2)=0$

$x=-9$ or $x=2$

g $x(3x+1)=0$

$x=0$ or $x=-\frac{1}{3}$

h $(x+5)(x-2)=0$

$x=-5$ or $x=2$

i $(x+5)(x-2)=0$

$x=-5$ or $x=2$

j $2(3x+5)(3x-5)=0$

$x=-\frac{5}{3}$ or $x=\frac{5}{3}$

k $(x+4)(x+5)=0$

$x=-4$ or $x=-5$

l $(x-6)(3x+4)=0$

$x=6$ or $x=-\frac{4}{3}$

2 **a** $2x(2x-5)=0$

$x=0$ or $x=\frac{5}{2}$

b $(x-4)(x-2)=0$

$x=4$ or $x=2$

c $(x+5)(x-1)=0$

$x=-5$ or $x=1$

d $(x-6)(x+2)=0$

$x=6$ or $x=-2$

e $(x+7)(x-1)=0$

$x=-7$ or $x=1$

f $x(x+4)=0$

$x=0$ or $x=-4$

g $(2x+5)(2x-5)=0$

$x=-\frac{5}{2}$ or $x=\frac{5}{2}$

h $(x-8)(x-4)=0$

$x=8$ or $x=4$

i $2(x+5)(x-2)=0$

$x=-5$ or $x=2$

j $(x+10)(x-7)=0$

$x=-10$ or $x=7$

3 **a** $(x+5)(x-2)=0$

$x=-5$ or $x=2$

b $(x-7)(x+4)=0$

$x=7$ or $x=-4$

c $(x+5)(x-3)=0$

$x=-5$ or $x=3$

d $(x+10)(x-2)=0$

$x=-10$ or $x=2$

e $(x-2)(x+1)=0$

$x=2$ or $x=-1$

f $(x-4)(x-2)=0$

$x=4$ or $x=2$

Exercise 19C

1 **a** $a=3,\ b=2,\ c=-4$

b $a=4,\ b=0,\ c=-8$

c $a=1,\ b=5,\ c=-2$

d $a=-3,\ b=4,\ c=2$

e $a=-7,\ b=4,\ c=0$

f $a=-4,\ b=-3,\ c=12$

g $a=3,\ b=2,\ c=-7$

h $a=2,\ b=-3,\ c=5$

i $a=2,\ b=6,\ c=-3$

j $a=1,\ b=-4,\ c=9$

k $a=5,\ b=-10,\ c=-4$

l $a=3,\ b=-12,\ c=-9$

2 **a** $a=1,\ b=3,\ c=-1$

$x=0.3$ or $x=3.3$

b $a=2,\ b=4,\ c=-3$

$x=0.6$ or $x=-2.6$

c $a=2,\ b=8,\ c=2$

$x=-0.3$ or $x=2.4$

d $a=1,\ b=-7,\ c=2$

$x=0.3$ or $x=6.7$

e $a = 1,\ b = 4,\ c = 1$
$x = -0.3$ or $x = -3.7$

f $a = 3,\ b = 0,\ c = -10$
$x = 1.8$ or $x = -1.8$

g $a = 2,\ b = 3,\ c = -1$
$x = 0.3$ or $x = -1.8$

h $a = -3,\ b = -2,\ c = 12$
$x = 1.7$ or $x = -2.4$

i $a = -3,\ b = 2,\ c = 2$
$x = -0.6$ or $x = 1.2$

3 **a** $a = 3,\ b = -5,\ c = 1$
$x = 1.4$ or $x = 0.23$

b $a = 1,\ b = -8,\ c = 7$
$x = 7.0$ or $x = 1.0$

c $a = 4,\ b = -12,\ c = 2$
$x = 2.8$ or $x = 0.18$

d $a = 1,\ b = 10,\ c = 18$
$x = -2.4$ or $x = -7.7$

e $a = 2,\ b = -7,\ c = -1$
$x = 3.7$ or $x = -0.14$

f $a = 2,\ b = -3,\ c = -2$
$x = 2.0$ or $x = -0.5$

g $a = 1,\ b = -7,\ c = -3$
$x = 7.4$ or $x = -0.41$

h $a = 2,\ b = -10,\ c = -5$
$x = 5.5$ or $x = -0.46$

Exercise 19D

1 **a** $b^2 - 4ac = 41$; this requires the quadratic formula (QF)

b $b^2 - 4ac = 16$; this does not require the QF

c $b^2 - 4ac = 52$; this requires the QF

d $b^2 - 4ac = 400$; this does not require the QF

e $b^2 - 4ac = 49$; this does not require the QF

f $b^2 - 4ac = 8$; this requires the QF

g $b^2 - 4ac = 40$; this requires the QF

h $b^2 - 4ac = 176$; this requires the QF

i $b^2 - 4ac = 24$; this requires the QF

Exercise 19E

1 **a** $(x - 6)(x - 4) = 0$
x-axis is cut at $x = 6$ and $x = 4$

b $(x - 5)(x + 2) = 0$
x-axis is cut at $x = 5$ and $x = -2$

c $(x + 5)(x - 5) = 0$
x-axis is cut at $x = -5$ and $x = 5$

d $(x + 6)(x - 2) = 0$
x-axis is cut at $x = -6$ and $x = 2$

e $(2x + 3)(x - 5) = 0$
x-axis is cut at $x = -\frac{3}{2}$ and $x = 5$

f $3x(x + 4) = 0$
x-axis is cut at $x = 0$ and $x = -4$

2 **a** $a = 1,\ b = -10,\ c = 1$
$x = 9.9$ and $x = 0.1$

b $a = 3,\ b = -3,\ c = -10$
$x = 3.1$ and $x = -1.6$

c $a = -5,\ b = 0,\ c = 12$
$x = 1.6$ and $x = -1.6$

d $a = 3,\ b = 5,\ c = 1$
$x = -0.2$ and $x = -1.4$

e $a = 2,\ b = -7,\ c = 4$
$x = 2.8$ and $x = 0.7$

f $a = -2,\ b = 4,\ c = 1$
$x = -0.2$ and $x = 2.2$

Exercise 19F

1 **a** $(x + 5)(x - 1) = 0$
intersection at $x = -5$ and $x = 1$
so coordinates are $(-5, 3)$ and $(1,3)$

b $(x - 6)(x - 2) = 0$
intersections at $x = 6$ and $x = 2$
so coordinates are $(6, 4)$ and $(2, 4)$

c $(x + 4)(3x - 2) = 0$
intersections at $x = -4$ and $x = \frac{2}{3}$
so coordinates are $(-4, 9)$ and $\left(\frac{2}{3}, 9\right)$

d intersections at $x = 1$ and $x = \frac{5}{2}$
so coordinates are (1, 5) $\left(\frac{5}{2}, 5\right)$

e $(x - 2)(x - 2) = 0$
intersection at $x = 2$ so coordinate is (2, 3)

2 **a** $(x - 3)(x - 2) = 0$
intersections at $x = 3$ and $x = 2$
so coordinates are (3, 3) and (2, 1)

b $a = 1,\ b = -1,\ c = -4$
intersections at $x = -1.6$ and $x = 2.6$ so coordinates are approximately $(-1.6, -0.2)$ and $(2.6, 8.2)$

c $(2x - 1)(x + 1) = 0$
intersections at $x = \frac{1}{2}$ and $x = -1$
so coordinates are $\left(\frac{1}{2}, \frac{7}{2}\right)$ and $(-1, 2)$

d $(2x - 1)(2x - 1) = 0$
intersection at $x = \frac{1}{2}$ so coordinate is $\left(\frac{1}{2}, -4\right)$

e $(3x - 2)(x - 2) = 0$
intersections at $x = \frac{2}{3}$ and $x = 2$
so coordinates are $\left(\frac{2}{3}, \frac{8}{3}\right)$ and $(2, -8)$

3 **a** $(x + 5)(x + 3) = 0$
intersections at $x = -5$ and $x = -3$
so coordinates are $(-5, -13)$ and $(-3, -9)$

b $(x + 2)(x - 2) = 0$
intersections at $x = -2$ and $x = 2$
so coordinates are $(-2, 5)$ and $(2, -3)$

c $(2x + 1)(x - 5) = 0$
intersections at $x = -\frac{1}{2}$ and $x = 5$
so coordinates are $\left(-\frac{1}{2}, 4\right)$ and (5, 15)

d $(x - 5)(x + 3) = 0$
intersections at $x = 5$ and $x = -3$
so coordinates are (5, 20) and $(-3, -4)$

e $x(x - 2) = 0$
intersections at $x = 0$ and $x = 2$
so coordinates are $(0, -2)$ and (2, 16)

Exercise 19G

1 $110 = t^2 - t$
$(t - 11)(t + 10) = 0$
Since t is > 0, $t = 11$ teams

2 $22 = t^2 - 4t + 1$
$(t - 7)(t + 3) = 0$
since $t > 0$, $t = 7$ seconds

3 $36 = \frac{1}{2}n(n - 1)$
$(n - 9)(n + 8) = 0$
since $n > 1$, $n = 9$ people

4 **a** $x = y + 6$
$y = x - 6$, where y is Jim's age

b $(x - 6)x = 135$
$(x - 15)(x + 9) = 0$
since $x > 0$, $x = 15$ years and $y = 9$

5 Sarah's age $= x = 16$ years and so Dave's age is 20 years

6 **a** $x = 5$

b $x = 7$

7 $a = 5$

8 **a** $x = 5$

b area $= 72\text{ m}^2$

9 **a** $x = 5$

b area of square $= 64\text{ cm}^2$ and area of triangle $= 27\text{ cm}^2$

10 $x = 3$

11 **a** $a = 3$

b $a = 6$

12 **a** $b = 9 - 2x$

b $(12 - 2x)(9 - 2x) = 54$
$4x^2 - 42x + 54 = 0$
$2x^2 - 21x + 27 = 0$
$x = 9$ or $x = \frac{3}{2} x < 9$ so $x = \frac{3}{2}$

c $V = (12 - 2x)(9 - 2x)x$
$= 81\text{ cm}^3$

13 **a** $(3x + 1)(x + 5) = 2[2(x + 5 + 2) + 2(3x + 1)]$
$3x^2 + 16x + 5 = 2(8x + 16)$
$3x^2 + 16x + 5 = 16x + 32$
$3x^2 - 27 = 0$
$x^2 - 9 = 0$

b $x^2 - 9 = 0$
so $x = 3$
area of lawn $= (3x + 1)(x + 5)$
$= 10 \times 8 = 80\,\text{ft}^2$

c area of path $= 40\,\text{ft}^2$
$40 \times £3.50 = £140$

14 a $A_1 = 150 \times 100$
$A_2 = (150 - x)(100 - x)$
$A_2 = 0.75 \times A_1$
$0.75 \times 150 \times 100 = (150 - x)(100 - x)$
$11250 = 15000 - 250x + x^2$
$x^2 - 250x + 3750 = 0$

b $x = 16.028\text{m}$
New dimensions are
$(150 - 16) \times (100 - 16)$
$= 134\text{m} \times 84\text{m}$

15 a $2x + y = 16$ and $x^2 + y^2 = 52$

b $x = 6\text{m},\ y = 4\text{m}$

16 a $2x + 8 = 4y$
$x = 2y - 4$

b area of rectangle $= 4x = 4(2y - 4)$
$= 8y - 16$

c $y^2 = 4(2y - 4) + 9$
$y^2 - 8y + 7 = 0$

d $(y - 7)(y - 1) = 0$
$y = 1$ or $y = 7$
use $y = 7$ because $y = 1$ gives negative value for x
$y = 7$ gives $x = 10$
rectangle $= 10\,\text{cm} \times 4\,\text{cm}$
square $= 7\,\text{cm} \times 7\,\text{cm}$

Exercise 19H

1 a $a = 1,\ b = 6,\ c = 9$
$b^2 - 4ac = 0$: equal roots

b $a = 3,\ b = -4,\ c = 2$
$b^2 - 4ac = -8$: no real roots

c $a = 1,\ b = 5,\ c = -1$
$b^2 - 4ac = 29$: two real roots

d $a = 5,\ b = 4,\ c = 9$
$b^2 - 4ac = -164$: no real roots

e $a = -1,\ b = -2,\ c = 4$
$b^2 - 4ac = 20$: two real roots

f $a = 9,\ b = -6,\ c = 1$
$b^2 - 4ac = 0$: equal roots

2 a $a = 3,\ b = 4,\ c = 0$
$b^2 - 4ac = 16$: two points of contact

b $a = 1,\ b = \frac{1}{2},\ c = \frac{1}{4}$
$b^2 - 4ac = -\frac{3}{4}$: no points of contact

c $a = 2,\ b = 0,\ c = -9$
$b^2 - 4ac = 72$: two points of contact

d $a = 1,\ b = 4,\ c = 6$
$b^2 - 4ac = -8$: no points of contact

e $a = -1,\ b = 3,\ c = 0$
$b^2 - 4ac = 9$: two points of contact

f $a = 5,\ b = -4,\ c = -3$
$b^2 - 4ac = 76$: two points of contact

3 a $a = 1,\ b = 1,\ c = 3$
$b^2 - 4ac = -11$: no real roots

b $a = 1,\ b = -4,\ c = 1$
$b^2 - 4ac = 12$: two real roots

c $a = 1,\ b = -8,\ c = 16$
$b^2 - 4ac = -11$: two equal roots

d $a = 4,\ b = -7,\ c = -2$
$b^2 - 4ac = 81$: two real roots

e $a = 4,\ b = 4,\ c = 1$
$b^2 - 4ac = 0$: two equal roots

f $a = 1,\ b = 2,\ c = 1$
$b^2 - 4ac = 0$: two equal roots

4 $a = 1,\ b = 3,\ c = 5$
$b^2 - 4ac = -11$: no real roots

5 $a = 1,\ b = -4,\ c = -7$
$b^2 - 4ac = 44$: two real roots

6 $k \leq 2$

7 $p \leq 1$

8 $k = \pm 5$

9 $x^2 - 3x + 2 = 2x - 9$

$x^2 - 5x + 11 = 0$

$b^2 - 4ac = -19$ so no real solution.

10 a $k = 25$ or $k = 1$

b $k = 0$ or $k = 5$

Activity pp. 201–203

1 a $a = 1,\ b = 1,\ c = 1$

b $a = 3,\ b = 0,\ c = 1$

c $a = 7,\ b = 2,\ c = -1$

2 a

n	1	2	3	4	5
R	2	4	7	11	16

$a = \frac{1}{2},\ b = \frac{1}{2},\ c = 1$

$R = \frac{1}{2}n^2 + \frac{1}{2}n + 1$

b $\frac{1}{2} \times 15^2 + \frac{1}{2} \times 15 + 1 = 121$

c $46 = \frac{1}{2}n^2 + \frac{1}{2}n + 1$

$n = 9$

d $35 = \frac{1}{2}n^2 + \frac{1}{2}n + 1$

$b^2 - 4ac = 1^2 - 4 \times 1 \times (-68) = 273$

This has no integer root.

Chapter 20

Exercise 20A

1 a $x = 12.37$

b $x = 12.45$

c $x = 5.68$

d $x = 24.44$

2 Poles are 6.13 m apart

3 Q and R are 7.61 km apart

4 PQ is 5.83 unit

Exercise 20B

1 **a**, **b**, **d**, and e are right-angled; **c** and **f** are not right-angled

2 **a** and **b** are not rectangular;

c is rectangular

3 Shelf is not at right angles to the wall.

4 Miranda

5 Neither angle is a right angle so the QA department will not send the bracket to the aeroplane fitters.

Exercise 20C

1 a 10.25 cm

b 13.60 m

c 13.34 cm

2 a 57.45 m

b 8.85 m

3 23.69 cm

4 2.33 m

5 Space diagonal = 2.92 m, so the pipe will fit

Exercise 20D

1 a $AB = 4.24$ units

b $CD = 5.39$ units

c $EF = 5.10$ units

d $GH = 10.5$ units

2 $PQ = 5.92$ units

3 a $D(4, 3, 5)$

b $AD = 7.07$ units

4 a $P(6, 3, 7)$

b $MP = 8.19$ units

5 a $K(6, 5, 6)$

b $AK = 4.69$ units

Chapter 21

Exercise 21A

1 a $a° = 16°$

b $b° = 111°$

c $c° = 66°$

d $d° = 38°$

e $e° = 118°$

f $f° = 85°$

2 a $a° = 136°$

b $b° = 127°$

c $c° = 111°$

d $d° = 22°$

Exercise 21B

1 STQ = 41°
2 BOD = 130°
3 QNP = 100°
4 WXY = 34°
5 PQO = 37°
6 YXW = 36°
7 OCB = 72°
8 RQT = 15°

Exercise 21C

1 a x = 19.94 cm
b x = 5 cm
c x = 1 cm
d x = 19.93 cm
e x = 5.37 cm
f x = 13.42 m

2 a x = 6.53 cm
b x = 6.58 m
c x = 13.16 cm
d x = 23.05 m

Exercise 21D

1 AB = 114.9 cm
2 h = 6.65 m
3 Ship B passes within 33.17 km, so alarm will not be activated.
4 w = 0.872 m
5 w = 20.4 cm
6 h = 27.80 cm
7 w = 114.89 cm
8 d = 8.62 cm
9 a OPQ = 30°
b OP = 1.732 cm
10 AOB = 97.18°

Activity p. 233

1 a

Name of polygon	Number of sides	Sum of the interior angles
Quadrilateral	4	2 × 180 = 360°
Pentagon	5	3 × 180 = 540°
Hexagon	6	4 × 180 = 720°
Heptagon	7	5 × 180 = 900°
Octagon	8	6 × 180 = 1080°
Nonagon	9	7 × 180 = 1260°
Decagon	10	8 × 180 = 1440°
Dodecagon	12	10 × 180 = 1800°

b $s = (n - 2) \times 180$

2 s = 360°

Activity p. 234

1

Regular polygon	Number of sides	Angle at the centre	Interior angle	Exterior angle
Square	4	$\frac{360}{4} = 90°$	90°	90°
Pentagon	5	$\frac{360}{5} = 72°$	108°	72°
Hexagon	6	$\frac{360}{6} = 60°$	120°	60°
Octagon	8	$\frac{360}{8} = 45°$	135°	45°

2 $E = \frac{360}{n}$

3 $I = 180 - \frac{360}{n}$

4 $n = 24$

Exercise 21E

1 a $I = 156°, E = 24°$

b $I = 160°, E = 20°$

c $I = 175°, E = 5°$

2 a $S = 1800°$

b $S = 2160°$

c $S = 3600°$

3 a $x° = 63°$

b $x° = 153°$

c $x° = 131°$

4 $x° = 281°$

5 9 sides

6 10 sides

Chapter 22

Exercise 22A

1 a $x = 10.29$ cm

b $x = 6.00$ cm

c $x = 8.17$ m

d $x = 13.5$ cm

e $x = 5.50$

f $x = 3.27$ m

2 a $x = 5.6$ mm

b $x = 6.29$ m

c $x = 4.8$ cm

d $x = 5.45$ m

3 a $x = 7.00$ m

b $x = 6.67$ cm

c $x = 14.88$ m

d $x = 7.69$ cm

4 length = 87.0 cm

5 Jake is 1.25 m tall.

6 $AD = 2.40$ m

7 House is 5.00 m high.

8 $AB = 17.3$ m

Activity p. 241

Pupil's own answers.

Exercise 22B

1 800 cm^2

2 4302 cm^2

3 590 ml

4 113 litres

5 62 cm^2

6 £48.83

7 £125

8 £127

9 a £3.84

b The cost is proportional to the volume because $\left(\frac{25}{15}\right)^3 \times £1.62 = £7.50$

10 No, the large box is over-priced because $\left(\frac{8}{6}\right)^3 \times £2.40 = £5.69$

Exercise 22C

1 length = 10.1 cm

2 $h = 12.0$ cm

3 height = 13.7 cm

4 area = 28.0 cm^2

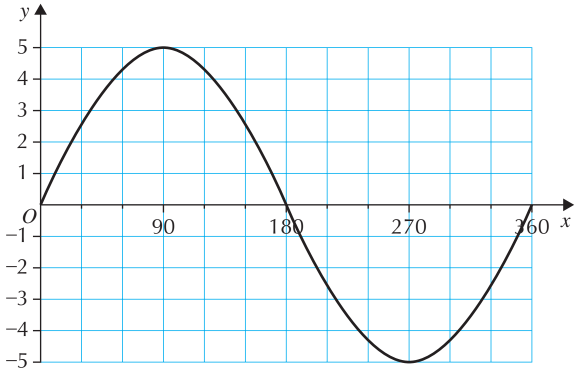
5 volume = 912.9 cm^3

6 area = 4.6 cm^2

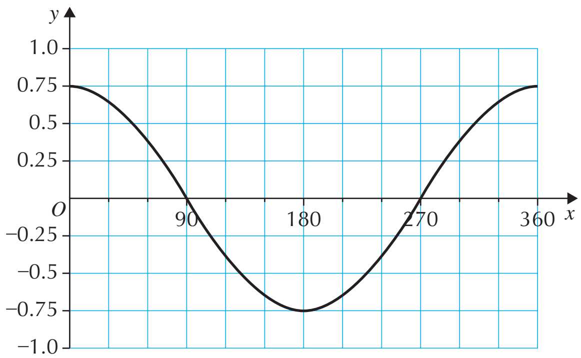
Chapter 23

Exercise 23A

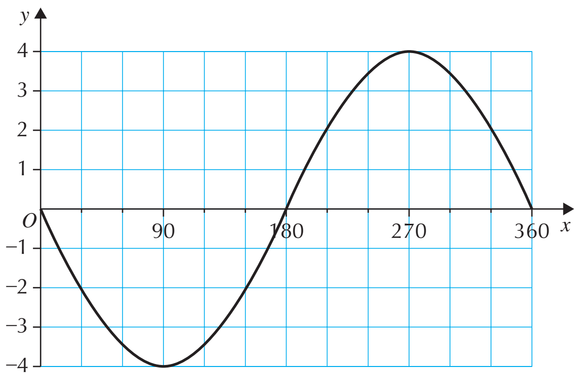
1 a



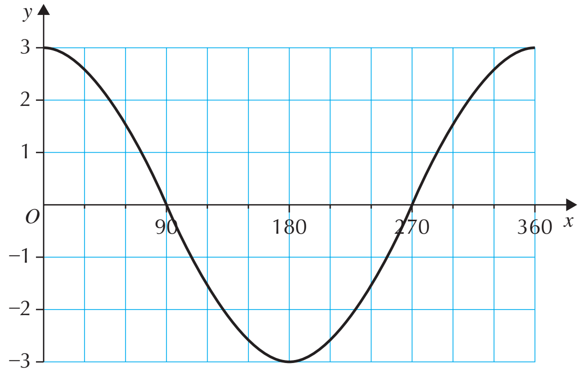
b



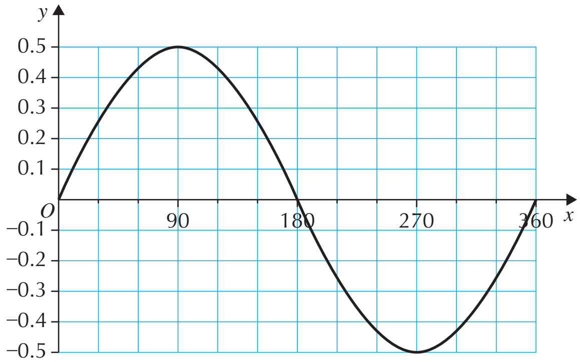
c



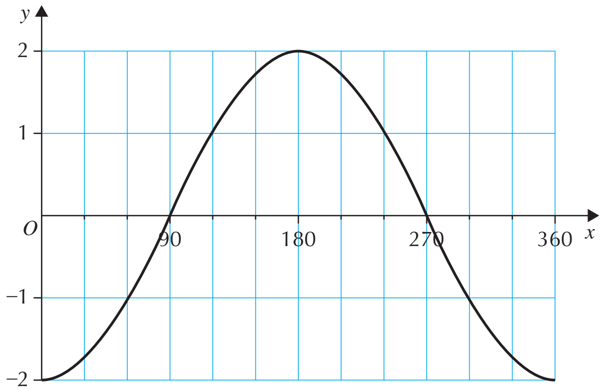
d



e



f



2 **a** $4\cos x$

b $2\sin x$

c $-3\sin x$

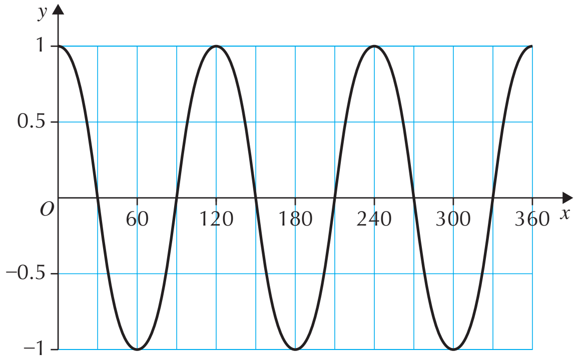
d $0.5\cos x$

e $-6\cos x$

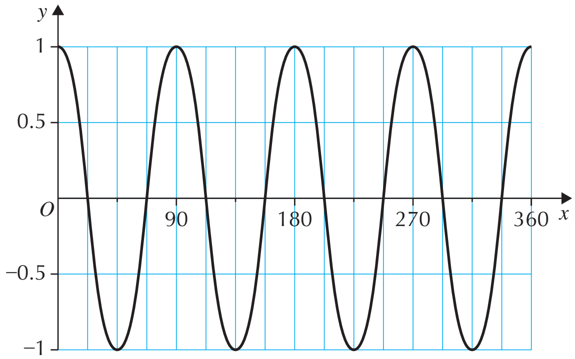
f $0.25\sin x$

Exercise 23B

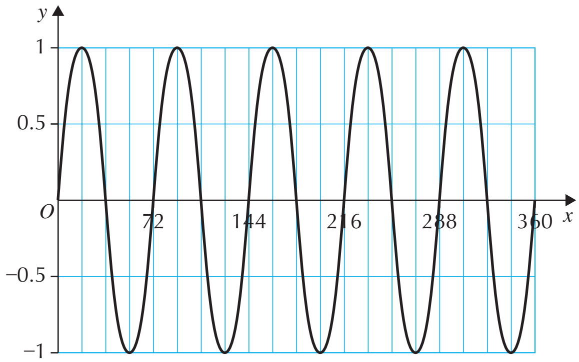
1 a



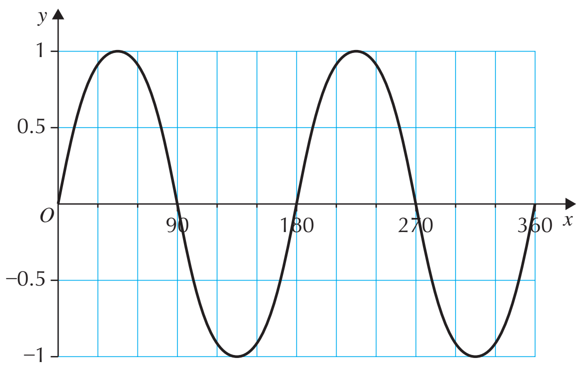
b



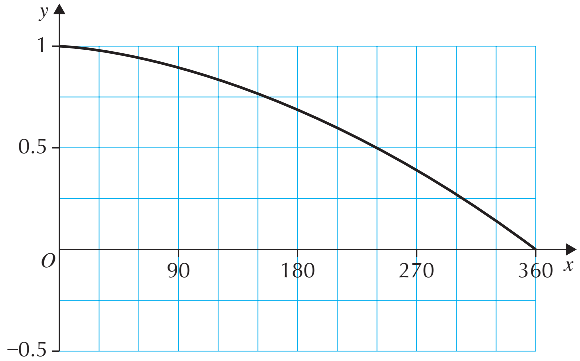
c



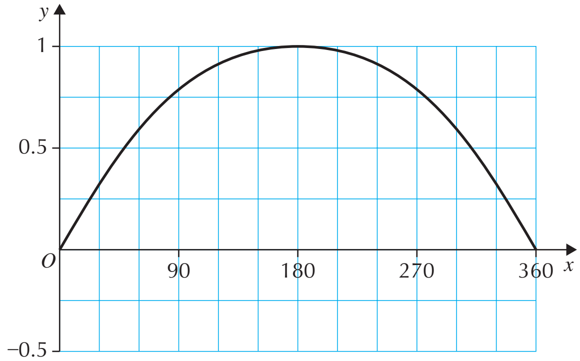
d



e



f



2 a $\sin 2x$

b $\sin 3x$

c $\cos 1.5x$

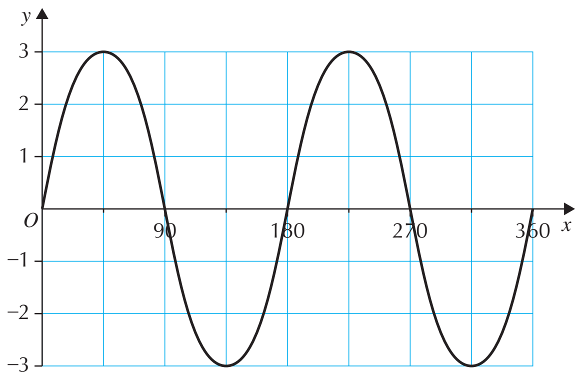
d $\cos 4x$

e $-\sin x$

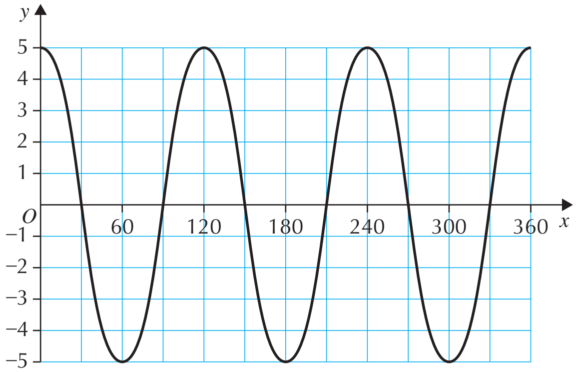
f $-\cos 3x$

Exercise 23C

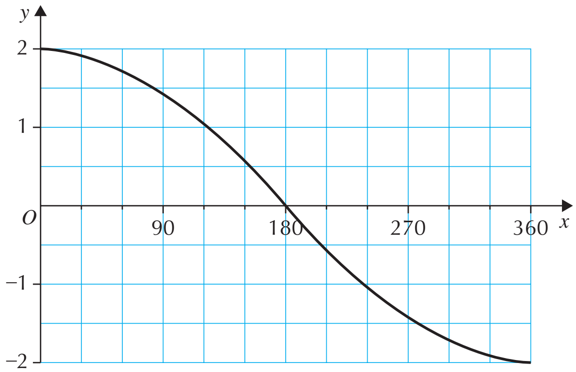
1 a



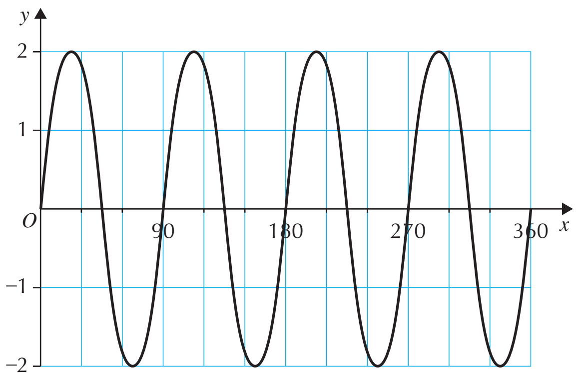
b


y
5
4
3
2
1
O
−1
−2
−3
−4
−5
60
120
180
240
300
360
x

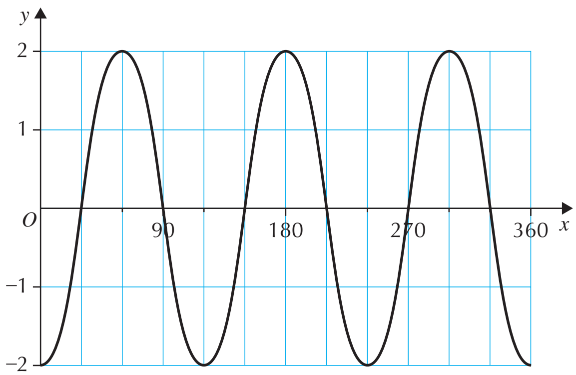
c


y
2
1
O
−1
−2
90
180
270
360
x

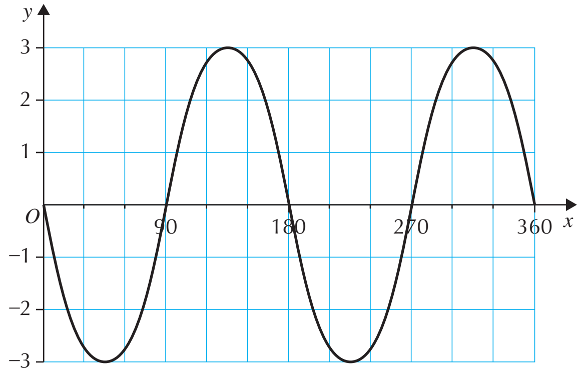
d


y
2
1
O
−1
−2
90
180
270
360
x

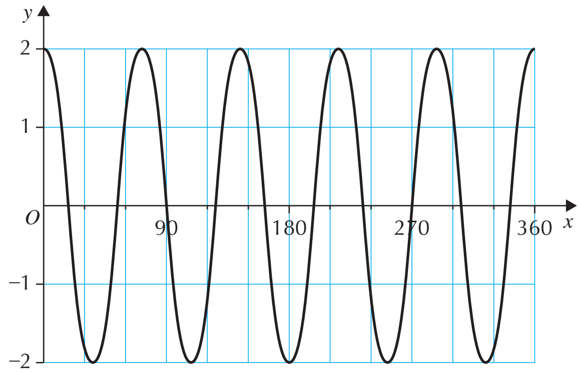
e


y
2
1
O
−1
−2
90
180
270
360
x

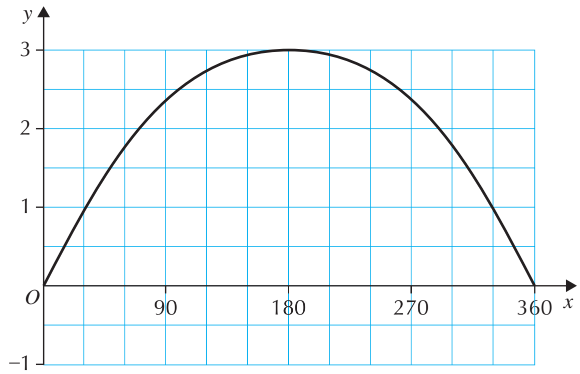
f


y
3
2
1
O
−1
−2
−3
90
180
270
360
x

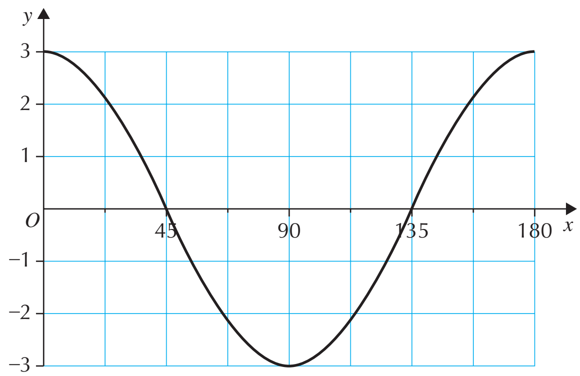
g


y
2
1
O
−1
−2
90
180
270
360
x

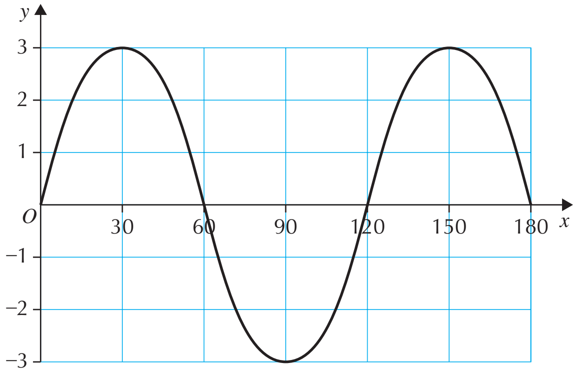
h


y
3
2
1
O
−1
90
180
270
360
x

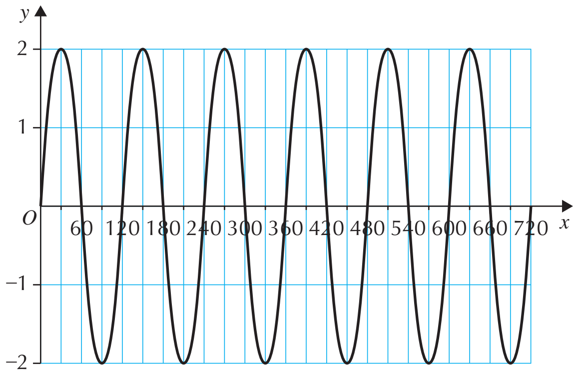
2 a


y
3
2
1
O
−1
−2
−3
45
90
135
180
x

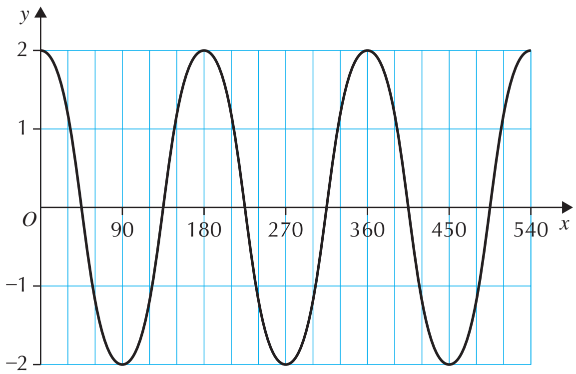
b



c



d



3 **a** $3\cos 3x$

b $2\sin 4x$

c $4\sin 2x$

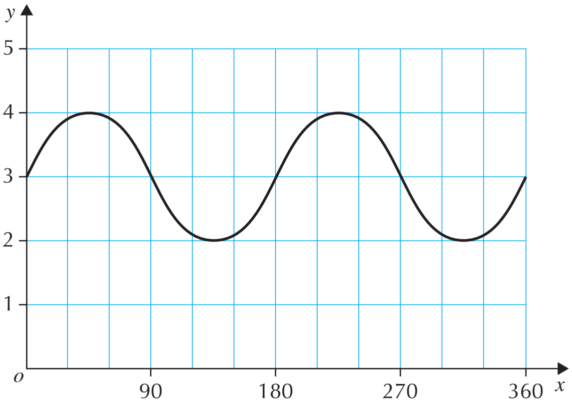
d $-3\sin 2x$

e $0.5\cos 9x$

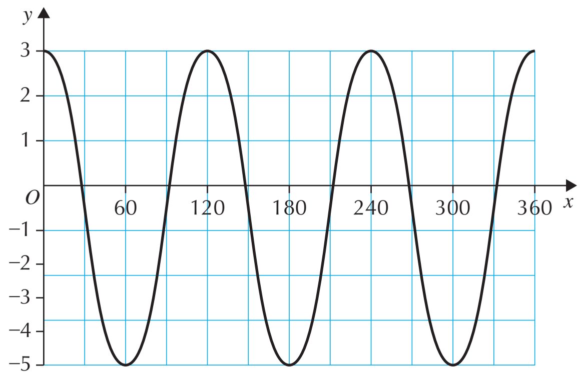
f $5\sin 0.5x$

Exercise 23D

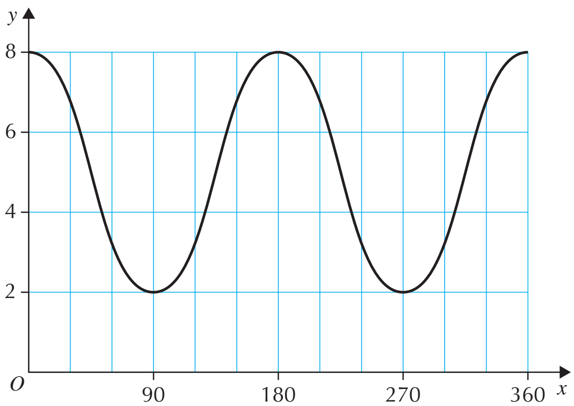
1 **a**



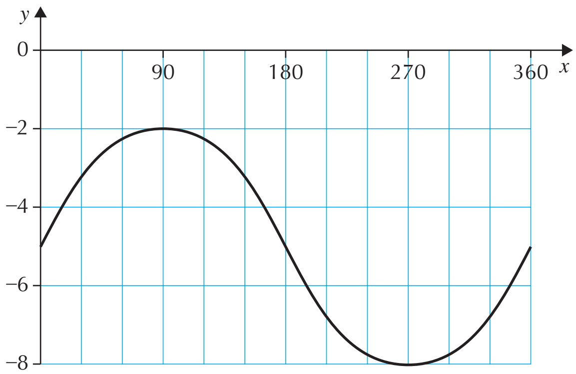
b



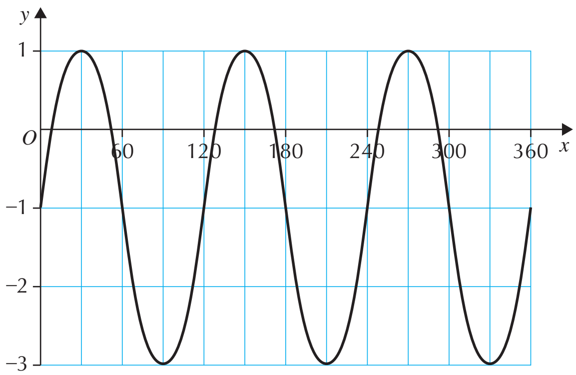
c



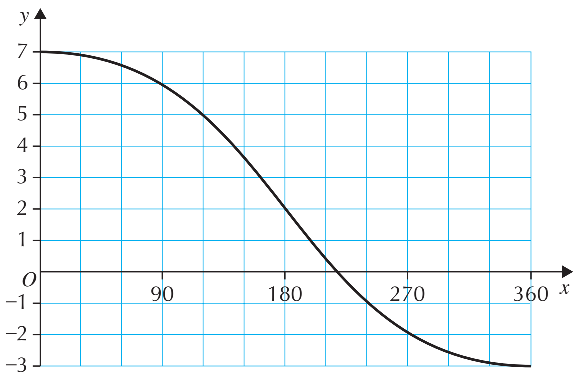
d


y
0
−2
−4
−6
−8
90
180
270
360
x

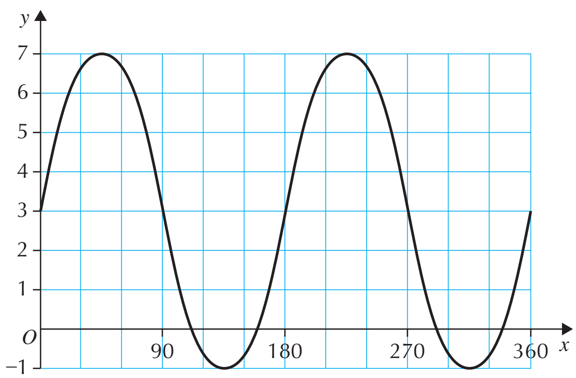
e


y
1
O
−1
−2
−3
60
120
180
240
300
360
x

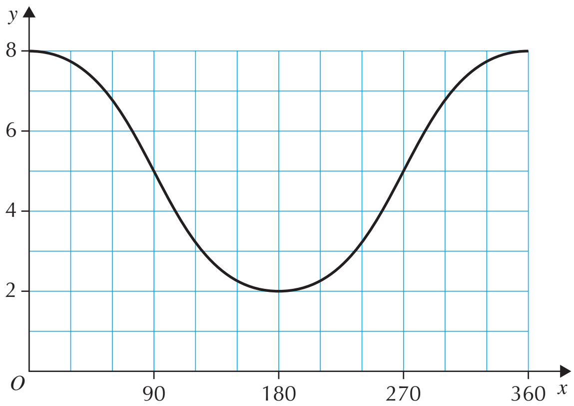
f


y
7
6
5
4
3
2
1
O
−1
−2
−3
90
180
270
360
x

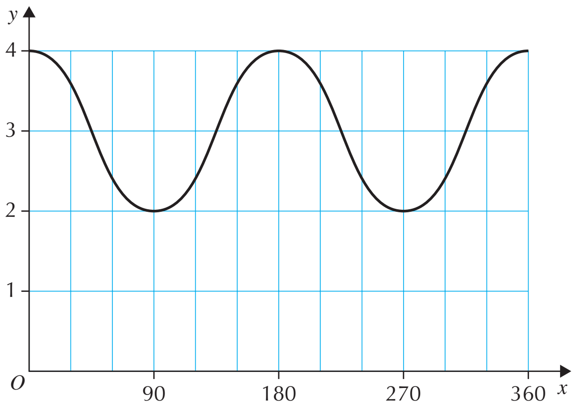
g


y
7
6
5
4
3
2
1
O
−1
90
180
270
360
x

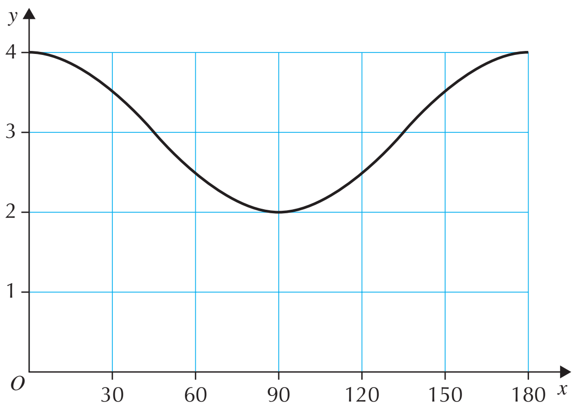
h


y
8
6
4
2
O
90
180
270
360
x

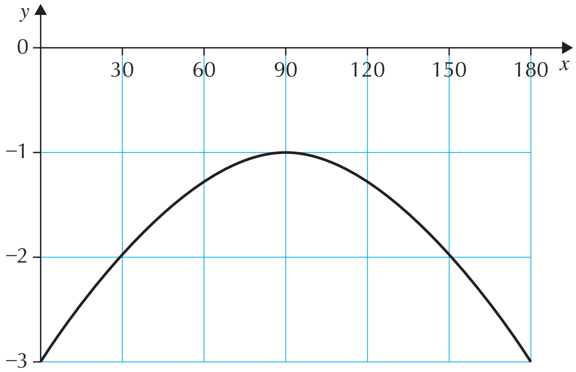
i


y
4
3
2
1
O
90
180
270
360
x

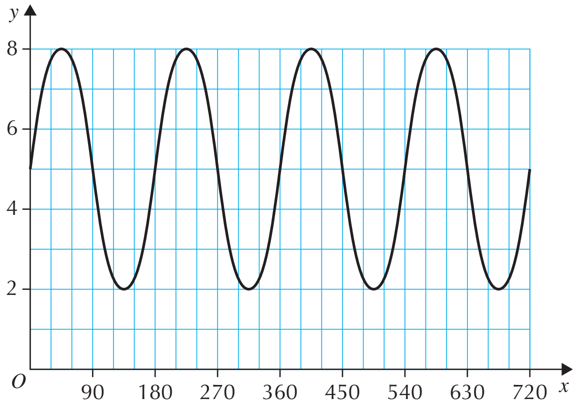
2 a


y
4
3
2
1
O
30
60
90
120
150
180
x

b



c



3 a $1 + 6\sin 2x$

b $-1 + 4\cos 3x$

c $4 + 3\sin 4x$

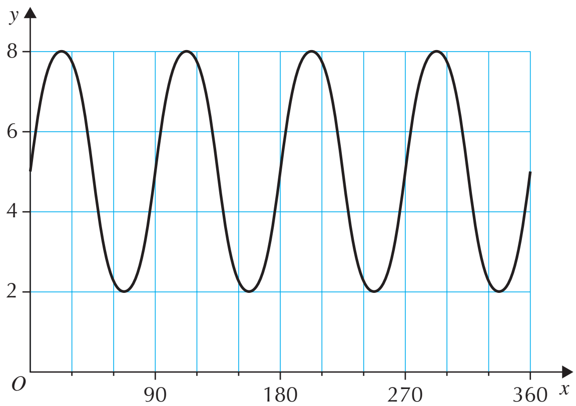
d $2 + 5\cos 0.5x$

e $-2 + 5\cos 6x$

f $-4 + 4\sin 3x$

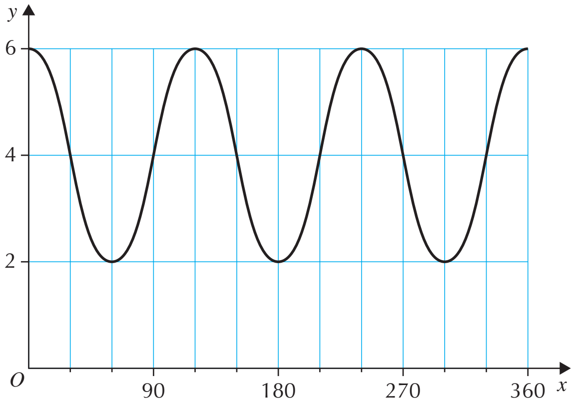
Exercise 23E

1



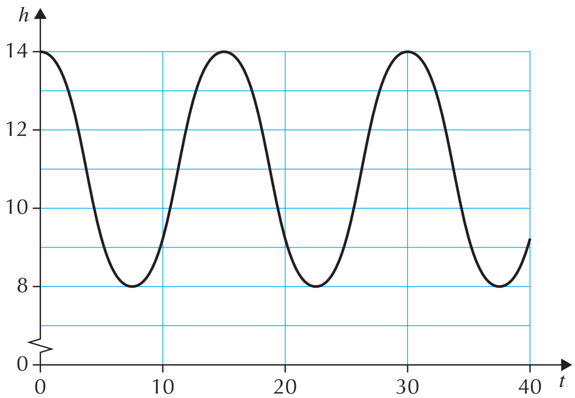
Max. value = 8.0 at x = 22.5°, 112.5°, 202.5°, 292.5°

2



Min. value = 2.0 at x = 60°, 180°, 300°

3 a

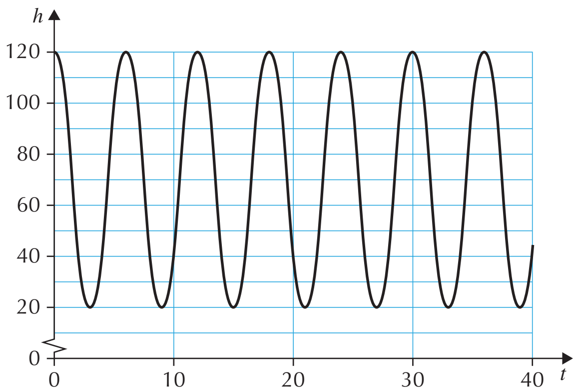


After 10 s h = 9.5 m

b Max. height h = 14 m at t = 0 s then t = 15 s

c Min. height h = 8 m at t = 7.5 s

4 a



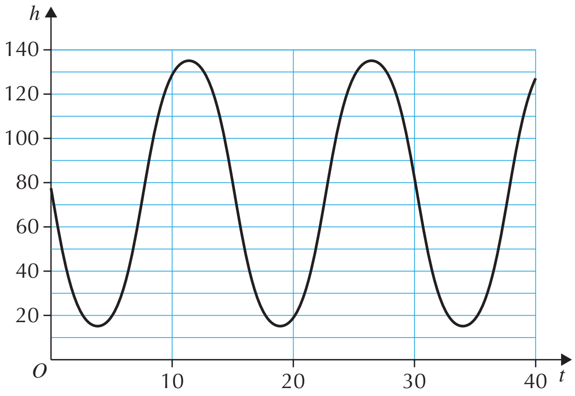
After 20 s h = 95 cm

b 6 s

c **i** Further from the ground $h = 120$ cm

ii This occurs at $t = 3$ s.

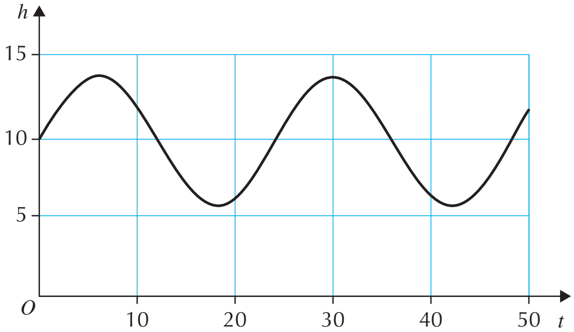
5 **a**



After 8 min, $h = 87.47$ m.

b Max. height at $t = 11.25$ min or 11 min 15 s

6 **a**



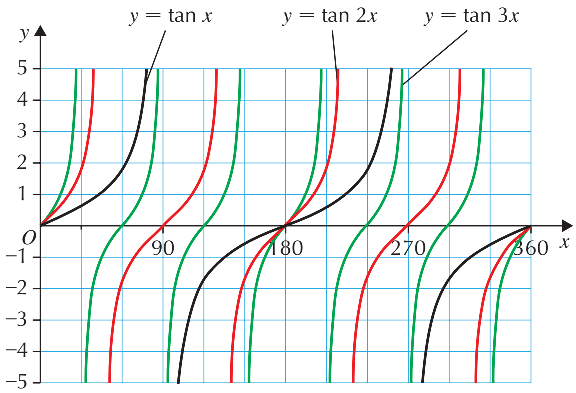
At 3 pm ($t = 9$), $h = 12.83$ m

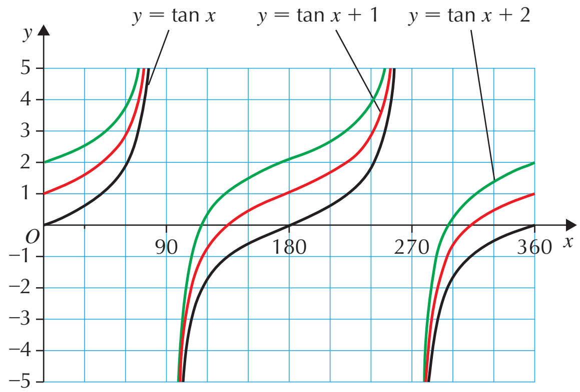
b Max. height at $t = 6$ hours or midday.

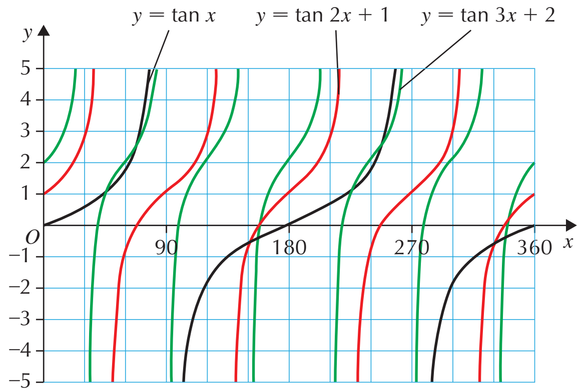
c Yes, water level will fall below 7 m between $t = 15.2$ hours and $t = 20.8$ hours, or 9 pm and 3 am.

Activity pp. 266–267

1

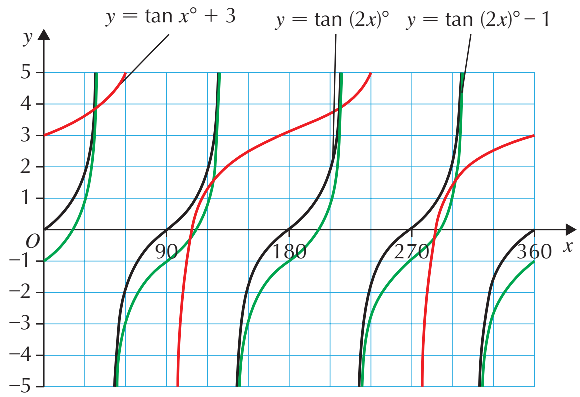






The diagrams show that increasing b results in more 'compressed' and higher frequency graphs. Increasing c simply raises the graph vertically by the amount c. Neither b nor c move the graph in the x-direction.

2



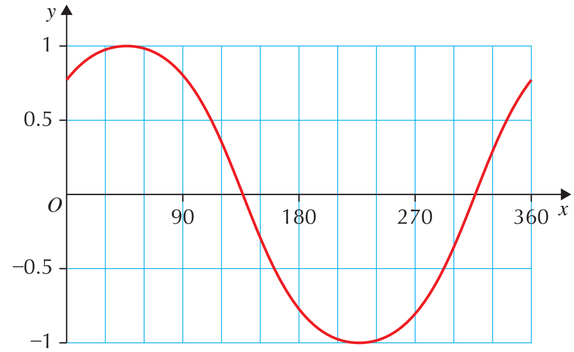
3 **a** $\tan x° + 2$

b $\tan 3x°$

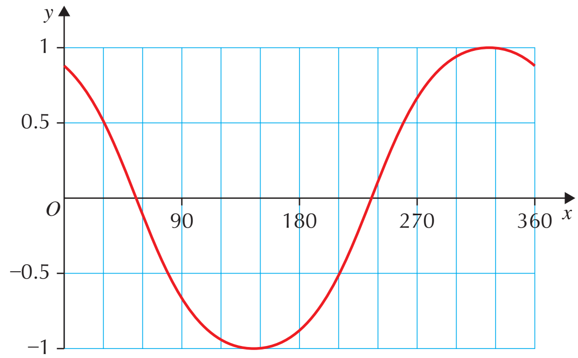
c $\tan 6x° - 1$

Exercise 23F

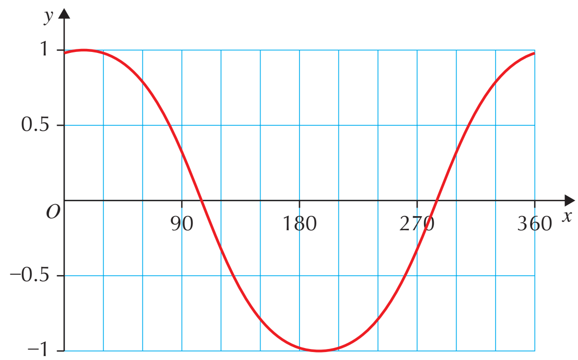
1 a



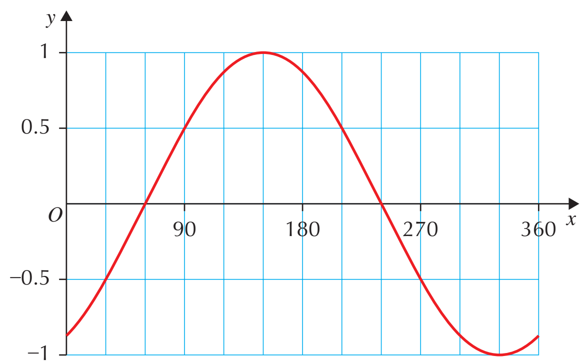
b



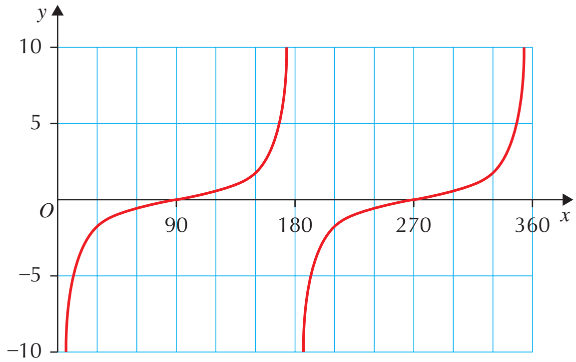
c



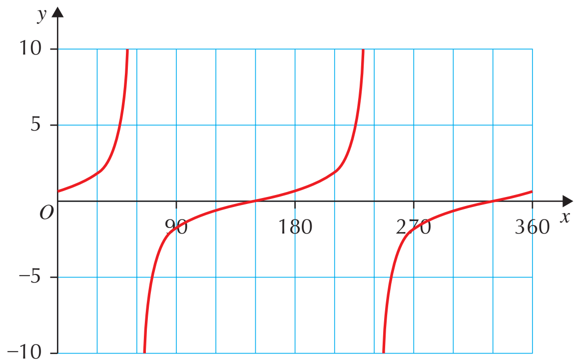
d



e



f



2 a $\sin(x - 30)^\circ$

b $\cos(x + 40)^\circ$

c $\sin(x - 30)^\circ$

d $\sin(x + 25)^\circ$

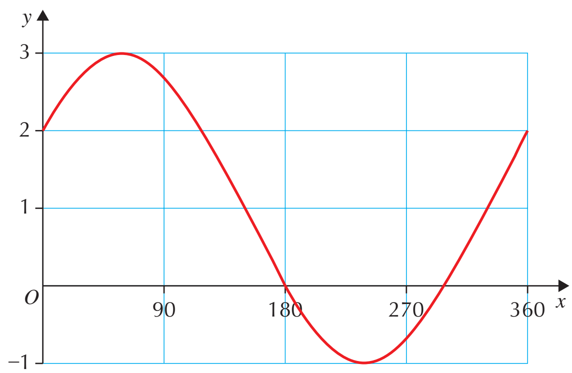
e $\sin(x + 20)^\circ$

f $\cos(x + 70)^\circ$

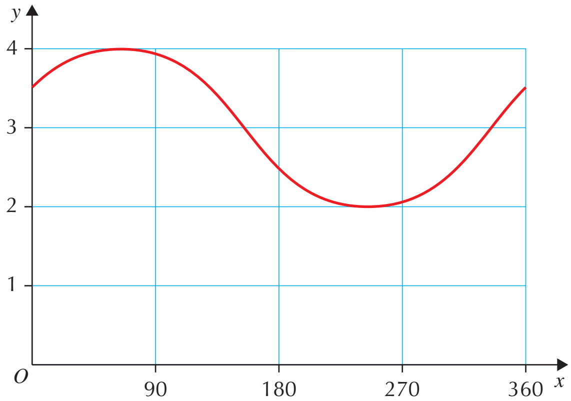
g $\tan(x + 30)^\circ$

h $\tan(x - 18)^\circ$

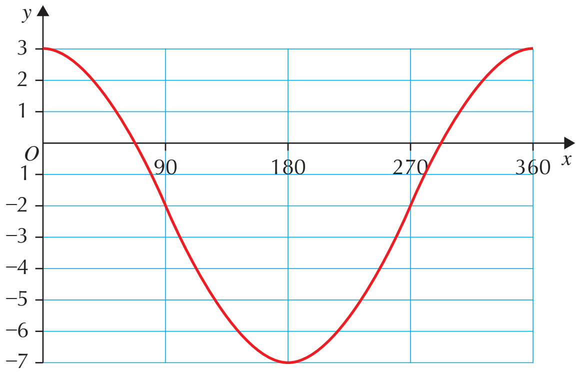
3 a



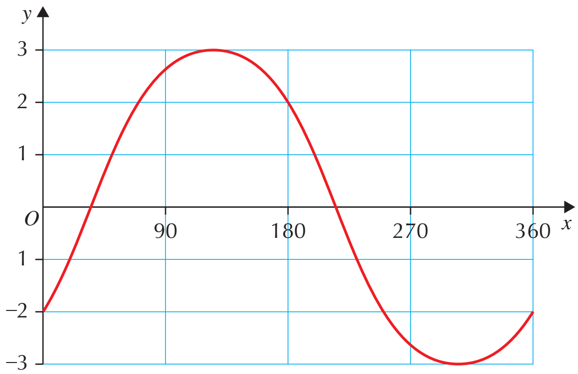
b



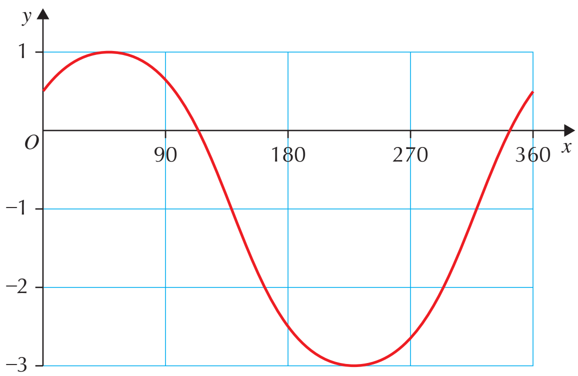
c



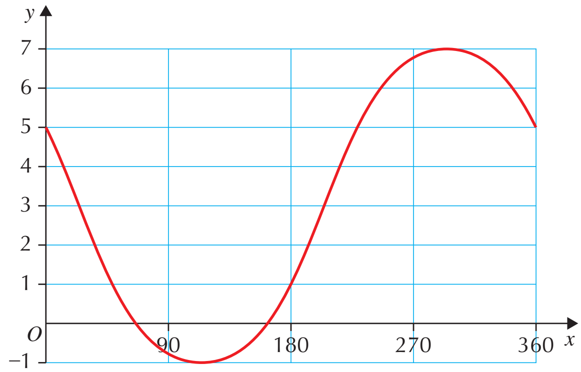
d



e



f



Activity p. 270

Function	Transformation
$y = \mathrm{f}(x) + c$	If c is +ve, the graph moves up c units If c is –ve, the graph moves down c units
$y = a\,\mathrm{f}(x)$	If a is > 1, graph stretches vertically, if < 1 graph compresses
$y = \mathrm{f}(bx)$	If b is > 1, graph stretches horizontally, if < 1 graph compresses
$y - \mathrm{f}(x + d)$	If d is + ve, graph slides left by a units, if d is –ve graph slides right

Chapter 24

Exercise 24A

1 **a** $\theta = 90$

b $\theta = 90°, 270°$

c $x = 0°, 180°, 360°$

d $x = 270$

e $x = 0°, 360°$

2 **a** $x = 0°, 180°, 360°, 540°, 720°$

b $x = 0°, 360°, 720°$

Exercise 24B

1 **a** –ve

b +ve

c –ve

d +ve

e +ve

f –ve

g –ve
h +ve
i +ve
j –ve
k +ve
l –ve

2 a 135°, 225°, 315°
b 120°, 240°, 300°
c 30°, 210°, 330°
d 40°, 220°, 320°
e 140°, 220°, 320°
f 35°, 145°, 325°
g 165°, 195°, 345°
h 10°, 170°, 190°
i 20°, 160°, 340°
j 65°, 245°, 295°
k 25°, 155°, 335°
l 15°, 165°, 195°

Exercise 24C

1 a $\theta = 17.5°$, $\theta = 162.5°$
b $\theta = 55.9°$, $\theta = 304.1°$
c $\theta = 71.6°$, $\theta = 251.6°$
d $\theta = 40.2°$, $\theta = 139.8°$
e $\theta = 77.5°$, $\theta = 257.5°$
f $\theta = 26.9°$, $\theta = 333.1°$
g $\theta = 13.0°$, $\theta = 193.0°$
h $\theta = 57.3°$, $\theta = 302.7°$

2 a $x = 110.0°$, $x = 250.1°$
b $x = 104.0°$, $x = 284.0°$
c $x = 228.6°$, $x = 311.4°$
d $x = 98.7°$, $x = 278.7°$
e $x = 207.1°$, $x = 332.9°$
f $x = 103.3°$, $x = 256.7°$
g $x = 198.9°$, $x = 341.1°$
h $x = 129.8°$, $x = 309.8°$

3 a $p = 30.00°$, $p = 150.00°$
b $p = 101.54°$, $p = 258.46°$
c $p = 106.39°$, $p = 286.39°$
d $p = 63.70°$, $p = 296.30°$
e $p = 189.79°$, $p = 350.21°$
f $p = 85.24°$, $p = 265.24°$
g $p = 64.16°$, $p = 115.84°$
h $p = 101.83°$, $p = 258.17°$

4 a $x = 37°$
b $x = 38°$, $x = 142°$, $x = 398°$, $x = 502°$
c $x = 256°$
d $x = 226°$, $x = 314°$, $x = 586°$, $x = 674°$
e $x = 123°$
f $x = 459$

Exercise 24D

1 a $x = 41.8°$, $\theta = 138.2°$
b $\theta = 75.5°$, $\theta = 284.5°$
c $\theta = 54.5°$, $\theta = 234.5°$
d $\theta = 138.6°$, $\theta = 221.4°$
e $\theta = 126.9°$, $\theta = 306.87°$
f $\theta = 70.5°$, $\theta = 289.5°$
g $\theta = 90.0°$
h $\theta = 33.6°$, $\theta = 326.4°$
i $\theta = 53.1°$, $\theta = 126.9°$
j $\theta = 76.0°$, $\theta = 256.0°$
k $\theta = 180.0°$
l $\theta = 228.6°$, $\theta = 311.4°$

2 a $h = 56.2$m
b i $x = 30.0$ s
ii $x = 150.0$ s

3 a $h = 6.36$m
b $h = 23.27$m

4 a $h = 19$m
b $h = 8.5$m
c $t = 124.9$mins, 235.2mins

Activity p. 280

1 a $t = 4.78$ s
b $h = 28.04$ m
c $R = 264.25$ m

2 $\theta = 12.2°$

3 $\theta = 15.0°$

Exercise 24E

1 **a** (44.4, 0.7) and (135.6, 0.7)
b (101.5, –0.2) and (258.5, –0.2)
c (71.6, 3) and (251.6, 3)
d (233.1, –0.8) and (306.9, –0.8)
e (30.0, 4) and (150, 4)
f (109.5, –3) and (250.5, –3)

2 **a** $p = 3$, $q = 3$
b (41.8, 5) and (138.2, 5)

3 **a** $a = 5$, $b = 1$, $c = 1$
b (113.6, –1) and (246.4, –1)
c (473.6, –1) and (606.4, –1)

Exercise 24F

1 **a** $\theta = 45°$ and $135°$
b $\theta = 30°$ and $210°$
c $\theta = 30°$ and $330°$
d $\theta = 120°$ and $300°$
e $\theta = 240°$ and $300°$
f $\theta = 120°$ and $240°$

2 **a** $\theta = 30°$ and $150°$
b $\theta = 45°$ and $225°$
c $\theta = 45°$ and $315°$
d $\theta = 120°$ and $300°$
e $\theta = 150°$ and $210°$
f $\theta = 225°$ and $315°$

Exercise 24G

1 $\sin x \tan x = \sin x \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos x}$

2 $\sin^3 x + \sin x \cos^2 x = \sin x(\sin^2 x + \cos^2 x)$
$= \sin x$

3 $3\sin^2 x + 3\cos^2 x = 3(\sin^2 x + \cos^2 x) = 3$

4 $\frac{\sin x}{\tan x} = \frac{\sin x}{\left(\frac{\sin x}{\cos x}\right)} = \frac{\sin x \cos x}{\sin x} = \cos x$

5 $5 - 5\cos^2 x = 5(1 - \cos^2 x) = 5\sin^2 x$

6 $\frac{\sin^2 x}{1-\cos^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$

7 $\frac{\sin x \cos x}{\cos^2 x} = \frac{\sin x}{\cos x} = \tan x$